Structural Closure and the Cosmological Misnomer: Admissibility, Expansion, and the Geometry of Closed Systems

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Abstract

Standard cosmology classifies the universe as "open," "flat," or "closed" by the curvature parameter of Friedmann–Lemaître–Robertson–Walker solutions. Those labels describe the geometry of solutions within a pre-assumed framework. They do not address the prior question of what structure is admissible in the first place.

This paper proves that the cosmological and structural uses of "open" and "closed" operate at different logical levels and cannot be identified. Structural closure — the requirement that every admissible datum be internally recoverable from the system's own relational structure, with no external scaffolding available even in principle — is not a geometric condition. It is a logical precondition for invariant physical content.

We establish the following results. Any theory requiring observables to be invariant under a symmetry group already realizes the quotient-semantic aspect of structural closure in its observable sector, but full structural closure is stronger and is not implied (Theorems 4.1 and 4.2). The freedom in the curvature parameter $k \in \{-1, 0, +1\}$ is a consequence of structural inputs the FLRW framework imports prior to its dynamical analysis, not of physical necessity (Theorem 5.1). When structural closure is enforced at the manifold stage, admissible geometry is sharply constrained: for compact connected simply connected orientable Riemannian 3-manifolds, the only admissible geometry is S^3 (Theorem 6.1).

The simple-connectivity hypothesis is essential at the frame-transitivity level, as shown by the round $\mathbb{RP}^3$ (Theorem 6.3); however, a strengthened gauge-reduced transport criterion restores simple connectivity and again yields S^3 (Theorems 6.5 and 6.6).

On the observational side, we prove that all standard expansion signatures — Hubble recession, cosmological redshift, and distance scaling — depend only on the scale factor $a(t)$ and are independent of whether the spatial section is compact, flat, or hyperbolic (Theorem 7.2). In a structurally closed universe, the only internally admissible content of "expansion" is intrinsic metric scaling; in the closed spherical case singled out by the geometric argument, this is the increase of the scale factor within S^3, with no external space to expand into (Theorems 7.5 and 8.6). The standard balloon analogy, which models the universe as a surface embedded in an ambient space, requires exactly the external structure that closure excludes and is therefore inadmissible (Theorems 8.4 and 8.5).

Accordingly, the cosmological term "open" refers to a class of solutions within a pre-imposed framework, whereas "closed" in the structural sense refers to an admissibility condition on physical description.

1 Introduction

In standard cosmology, the spatial geometry of the universe is classified by the curvature parameter $k \in \{-1, 0, +1\}$ and the density parameter Ω. A universe with $\Omega < 1$ and $k = -1$ is termed "open": spatially non-compact and negatively curved. A universe with $\Omega > 1$ and $k = +1$ is termed "closed": spatially compact and positively curved. The case $\Omega = 1$ and $k = 0$ is "flat."

This classification is often taken as a statement about the nature of the universe. More precisely, it is a statement about the properties of solutions within a specific geometric model — the Friedmann–Lemaître–Robertson–Walker (FLRW) framework [8, 9, 10, 11] — which presupposes a spacetime manifold, a metric, a global time foliation, and an imposed comoving proper-time parameterization. The classification tells us which geometries are consistent with that model. It does not tell us which geometries are admissible in the first place.

This paper answers that prior admissibility question. It distinguishes two uses of "open" and "closed" that operate at entirely different logical levels. Cosmological "open" and "closed" are geometric properties of solutions. Structural "open" and "closed" concern the admissibility of physical content itself. These notions should therefore be kept distinct. The structural notion is logically prior to the cosmological classification.

The paper proceeds in three stages. First, it distinguishes structural closure from the FLRW curvature classification and shows why the latter does not answer the former. Second, at the manifold stage, it shows that frame-transitivity yields S^3 under the compactness, orientability, and simple-connectivity hypotheses, that the round $\mathbb{RP}^3$ is the residual obstruction when simple connectivity is dropped, and that a strengthened gauge-reduced transport condition restores the spherical conclusion. Third, it shows that the observational signatures of expansion depend only on the scale factor and therefore admit only an intrinsic interpretation in a structurally closed universe.

2 Two Notions of Open and Closed

2.1 Cosmological classification

The standard classification operates within the FLRW framework. The FLRW spacetime metric may be written

$$ds^2 = -dt^2 + a(t)^2 \, d\sigma^2, \tag{2.1}$$

where $a(t)$ is the scale factor and the constant-curvature spatial metric takes the form

$$d\sigma^2 = \frac{dr^2}{1 - kr^2} + r^2 \, d\Omega_2^2, \qquad k \in \{-1, 0, +1\}, \tag{2.2}$$

where $d\Omega_2^2$ is the round metric on S^2. In the simply connected textbook FLRW models, the curvature parameter k classifies the spatial geometry: $k = +1$ gives spherical geometry with spatial sections S^3, $k = 0$ gives flat geometry with spatial sections $\mathbb{R}^3$, and $k = -1$ gives hyperbolic geometry with spatial sections H^3.

In this setting, "open" means $k = -1$: the spatial sections are non-compact and negatively curved. For standard matter content satisfying the strong energy condition, such models expand indefinitely. "Closed" means $k = +1$: the spatial sections are compact and positively curved.

The key observation is that this entire classification presupposes the FLRW ansatz. The manifold, the metric signature, the product structure $M = \mathbb{R} \times \Sigma$, the homogeneity and isotropy of Σ, and the global time coordinate t are all imposed before the classification begins.

Remark 2.1 (What the FLRW classification assumes)**.** *The FLRW framework assumes: (i) a smooth spacetime manifold; (ii) a Lorentzian metric; (iii) a global time foliation $M = \mathbb{R} \times \Sigma$; (iv) spatial homogeneity and isotropy of each slice Σ_t; (v) an imposed comoving congruence and the associated proper-time foliation, often written using a cosmic time t. None of these is derived from a more primitive admissibility condition. Each is imported as structural input.*

2.2 Structural closure

Structural closure is a condition on the admissibility of physical content. It is not a property of solutions within a model; it is a constraint on what counts as a model in the first place.

Definition 2.2 (Structural closure)**.** *A physical system is* structurally closed *if every admissible physical datum is internally recoverable from the system's own relational structure. No external evaluator, reference frame, coordinate system, gauge section, time parameter, or background geometry is available even in principle.*

The formal content of structural closure is developed in [1]. We state the key consequences here.

Let U be a set of states equipped with a family $\mathcal{C}$ of binary comparison predicates $c : U \times U \to \{0, 1\}$. No topology, metric, or geometry is assumed. The intrinsic symmetry group is

$$G := \mathrm{Aut}(U, \mathcal{C}) = \big\{\phi \in \mathrm{Bij}(U) : c(\phi(u), \phi(v)) = c(u, v) \text{ for all } c \in \mathcal{C},\ u, v \in U\big\}. \tag{2.3}$$

A comparison world $(U, \mathcal{C})$ is *rectangularly complete* if the canonical factor map

$$\Theta : U \to X_A \times X_B, \qquad \Theta(u) = ([u]_\alpha, [u]_\beta), \tag{2.4}$$

is bijective, where $X_A := U/\alpha$ and $X_B := U/\beta$, and where α and β are the intrinsic congruences defined by left and right comparison profiles [1]. Rectangular completeness is the formal expression of structural closure at the level of comparison worlds. For the observable-sector discussion below, we write $X := U$.

Definition 2.3 (Coherent report)**.** *A map $R : X \to S$ is* coherent *if $R(g \cdot x) = R(x)$ for all $g \in G$ and all $x \in X$. Equivalently, R factors through the orbit projection $\pi : X \to \mathsf{Phys} := X/G$.*

Definition 2.4 (Admissibility under closure)**.** *Under structural closure, the admissible physical reports are exactly the coherent maps. A datum that does not factor through π depends on representational structure not internally recoverable, and is therefore inadmissible.*

The forced descent theorem [1] establishes that coherence and factorization through π are equivalent: a report is G-invariant if and only if it factors uniquely through the orbit projection. This is the universal property of the quotient, and it is the mathematical content of structural closure at the level of observables. To avoid level drift, we will use three terms consistently. Cosmological classification refers to the FLRW curvature label k and the geometry of model solutions. Quotient-semantic closure refers to the observable-sector statement that admissible reports factor through $\pi : X \to X/G$. Full structural closure refers to Definition 2.2 and its formal comparison-world realization via rectangular completeness; when enforced at the manifold stage, it constrains admissible geometry itself.

3 Logical Independence of the Two Notions

Proposition 3.1 (Cosmological classification does not determine structural closure)**.** *The cosmological classification of a universe as "open" or "closed" is logically independent of quotient-semantic closure and, by itself, does not determine full structural closure in the sense of Definition 2.2.*

Proof. We separate the observable-level quotient notion from the stronger framework-level structural notion.

Geometrically closed and structurally closed. S^3 with its round metric is compact ($k = +1$, cosmologically "closed") and, under the manifold-stage closure criterion of [2], its isometry group acts transitively on the orthonormal frame bundle, forcing all admissible reports to factor through the orbit projection. Hence S^3 satisfies structural closure at the manifold stage.

Geometrically open and quotient-semantically closed. Consider a system (X, G) in which every admissible report is G-invariant and factors through $\pi : X \to X/G$, but whose geometric realization has $k = -1$ spatial sections. At the quotient-semantic level — the level at which observables are classified — quotient-semantic closure is satisfied, even though the spatial geometry is hyperbolically curved and hence cosmologically "open." (Note: when structural closure is further enforced at the manifold stage, Theorem 6.1 excludes $k = -1$. The present example shows non-alignment at the observable level, before manifold-stage constraints are imposed.)

Geometrically closed and structurally open. Take S^3 but equip it with an external time parameter t not derived from internal relational data, and define a report $R(x, t) := f(t)$ that depends on this external parameter. The system is compact (cosmologically "closed") but admits a datum that does not factor through the internal orbit projection. Hence structural closure fails.

Geometrically open and structurally open. The standard FLRW model with $k = -1$ and an imposed comoving proper-time foliation is geometrically open and, by Remark 2.1, imports external structure. Hence structural closure fails.

From the second and fourth cases, cosmological classification does not determine whether quotient-semantic closure holds. From the first and third cases, even a cosmologically "closed" geometry does not by itself determine whether full structural closure holds. Since full structural closure is stronger than quotient-semantic closure, the cosmological labels do not determine structural closure in either sense. The two notions therefore belong to different logical levels and cannot be identified. □

Remark 3.2 (The conflation and its consequence)**.** *The word "closed" is used in cosmology to mean compact spatial sections. It is used in the structural framework to mean internal admissibility. These are different properties at different logical levels.*

Conflating them leads to the inference that observational evidence for $\Omega \leq 1$ constitutes evidence against a closed universe in the structural sense. Proposition 3.1 shows that cosmological labels do not support that inference.

4 Necessity of Quotient-Semantic Closure

The distinction drawn in Section 3 would be merely terminological if quotient-semantic closure were merely a modeling convenience. This section shows that, once physical observables are required to be invariant, admissible reports necessarily take quotient-semantic form.

Theorem 4.1 (Invariant observables imply quotient-semantic closure)**.** *Let (X, G) be a system with a group G acting on a state space X. If the physically meaningful reports on X are required to be G-invariant, then every such report factors through the orbit projection $\pi : X \to X/G$.*

Proof. Let $R : X \to S$ be a G-invariant map: $R(g \cdot x) = R(x)$ for all $g \in G$. By the universal property of the quotient, there exists a unique $\widetilde{R} : X/G \to S$ with $R = \widetilde{R} \circ \pi$. Hence R factors through π. □

Corollary 4.2 (Standard physics realizes quotient-semantic closure)**.** *Any physical framework that:*

(i) *admits a symmetry group G acting on its state space;*

(ii) *requires physical observables to be G-invariant;*

already realizes the quotient-semantic aspect of structural closure in its observable sector: admissible reports factor through the orbit projection.

Proof. Immediate from Theorem 4.1. □

Remark 4.3 (Quotient-semantic closure is not full structural closure)**.** *Corollary 4.2 does not say that standard frameworks are fully structurally closed. It says that their admissible observables already take quotient form. Full structural closure (Definition 2.2) is stronger: it requires that no external evaluator, background manifold, imposed foliation, or non-internal subsystem cut be available even in principle. Standard physics routinely imports such structure — background manifolds in general relativity, external time parameters in canonical quantum gravity, imposed Hilbert spaces in quantum mechanics — while simultaneously requiring observables to be invariant. The gap between quotient-semantic closure of the observable sector and full structural closure of the framework is precisely the gap that the four-axis taxonomy of [3] classifies.*

Proposition 4.4 (Minimality of structural closure)**.** *Structural closure — equivalently, rectangular completeness at the comparison-world level [1] — is the minimal condition that uniformly determines the quotient-semantic structure* $\pi : X \to \mathsf{Phys} = X/G$ *from intrinsic relational data. No strictly weaker condition suffices.*

Proof. This is the minimality theorem of [1]: any property of comparison worlds that uniformly determines the canonical product decomposition $U \cong X_A \times X_B$ compatible with the intrinsic congruences implies rectangular completeness. □

5 Origin of "Open Universe" Models

Theorem 4.1 raises a natural question: if standard invariance requirements already force observables into quotient form, why do cosmological models classified as "open" exist at all? The answer is that quotient-semantic closure of the observable sector is not full structural closure of the framework.

Proposition 5.1 (Open cosmological models arise from non-closure-enforcing frameworks)**.** *The FLRW framework admits solutions with* $k \in \{-1, 0, +1\}$ *because it imports structural inputs not determined by internal admissibility. Specifically:*

(i) *the smooth manifold* M *is assumed, not derived;*

(ii) *the Lorentzian metric* g *is assumed, not derived;*

(iii) *the global time foliation* $M = \mathbb{R} \times \Sigma$ *is imposed externally;*

(iv) *the spatial homogeneity and isotropy of* Σ *are imposed as symmetry assumptions, not derived from a minimal closure condition;*

(v) *the comoving proper-time foliation is imposed, not recovered from internal relational data.*

The freedom in k *is a consequence of these imported inputs.*

Proof. The FLRW spacetime metric (2.1), whose spatial part is (2.2), introduces the scale factor $a(t)$ and the curvature label k within an ansatz that already assumes a manifold, metric type, foliation, and symmetry. Einstein's field equations then constrain $a(t)$ once matter content and initial data are specified, but they do not derive the ansatz itself from an internal admissibility condition. Hence the availability of the $k \in \{-1, 0, +1\}$ model classes — and in particular the existence of $k = -1$ ("open") solutions — is a consequence of structural inputs supplied prior to the dynamical analysis, not of physical necessity. □

Remark 5.2 (The inversion). *Standard cosmology asks: given the FLRW framework, which value of k does the universe have? The structural question inverts this: given that the universe is a closed system, which frameworks are admissible? The FLRW classification is an answer to the first question. Theorem 5.1 shows that it cannot answer the second, because the framework itself does not enforce the closure condition that would constrain k.*

6 Geometric Consequences of Structural Closure

When structural closure is enforced not merely at the level of observables but at the geometric level — as a condition on the admissibility of the manifold arena itself — the freedom in k is sharply constrained. Under the frame-transitivity criterion, the spherical conclusion holds with an explicit simple-connectivity hypothesis; under the strengthened gauge-reduced criterion introduced below, the residual topological obstruction is removed.

Theorem 6.1 (Closure forces spherical geometry). *Let M be a compact connected simply connected orientable Riemannian 3-manifold. If M satisfies structural closure at the manifold stage — that is, if the isometry group $\mathrm{Iso}(M)$ is required to act transitively on the orthonormal frame bundle FM as a consequence of admissible transport between all frame configurations — then M is diffeomorphic to S^3.*

Proof. By the closed-system admissibility criterion [1], admissible transport between frame configurations is realized exactly by isometries. Therefore structural closure at the manifold stage forces $\mathrm{Iso}(M)$ to act transitively on the orthonormal frame bundle FM.

By Kobayashi's theorem [12], transitivity of $\mathrm{Iso}(M)$ on FM implies that M has constant sectional curvature. Hence M is a 3-dimensional space form.

Because M is compact and simply connected, only the spherical space form can occur. Indeed, a compact simply connected 3-manifold of constant sectional curvature is isometric to S^3/Γ, $\mathbb{R}^3/\Gamma$, or H^3/Γ for a discrete group Γ acting freely [7]. Compactness excludes the flat and hyperbolic cases, and simple connectivity forces $\Gamma = \{e\}$ in the spherical case. Therefore $M \cong S^3$. □

Corollary 6.2 (Closure eliminates the freedom in k). *Under the hypotheses of Theorem 6.1, the curvature parameter is no longer free: the only admissible case is $k = +1$. The "open" ($k = -1$) and "flat" ($k = 0$) cases are excluded.*

Proof. Immediate from Theorem 6.1: the only admissible geometry is S^3, which has constant positive curvature. □

Proposition 6.3 (The simple-connectivity hypothesis is essential). *The simple-connectivity hypothesis in Theorem 6.1 cannot be dropped under the current frame-transitivity criterion. Equipped with the metric induced from the round S^3, the* *manifold* $\mathbb{RP}^3$ *is compact, connected, orientable, not simply connected, and* $\mathrm{Iso}(\mathbb{RP}^3)$ *acts transitively on the orthonormal frame bundle* $F(\mathbb{RP}^3)$.

Proof. Let $M := \mathbb{RP}^3$ with the metric induced from the round S^3. Intrinsically, this is the compact Lie group SO(3) endowed with its corresponding bi-invariant metric. Hence M is compact, connected, and orientable; also $\pi_1(M) \cong \mathbb{Z}/2\mathbb{Z}$, so M is not simply connected.

For any $a, b \in \mathrm{SO}(3)$, the map

$$x \longmapsto axb^{-1}$$

is an isometry, because left and right translations preserve a bi-invariant metric. Therefore $\mathrm{SO}(3) \times \mathrm{SO}(3)$ acts transitively on M by isometries.

Fix the identity element $e \in M$. For each $a \in \mathrm{SO}(3)$, conjugation

$$C_a(x) := axa^{-1}$$

is an isometry fixing e. Its differential at e is the adjoint action $d(C_a)_e = \mathrm{Ad}_a$ on $T_eM \cong \mathfrak{so}(3)$. Under the standard identification $\mathfrak{so}(3) \cong \mathbb{R}^3$, the maps Ad_a are exactly the rotations in SO(3). Hence the isotropy at e acts transitively on the oriented orthonormal frames at e.

Now inversion

$$\iota(x) := x^{-1}$$

is an isometry of every bi-invariant metric. It fixes e and satisfies $d\iota_e = -\mathrm{id}$. Since $\dim M = 3$, this reverses orientation. Therefore the isotropy at e acts on T_eM by all of $O(3)$, not just by SO(3). So the isotropy is transitive on the full orthonormal frame space at e.

Combining point-transitivity with frame-transitivity at one point shows that $\mathrm{Iso}(M)$ acts transitively on the full orthonormal frame bundle FM. Thus $\mathbb{RP}^3$ satisfies the frame-transitivity condition used in Theorem 6.1 while failing simple connectivity. The simple-connectivity hypothesis is therefore essential under the current criterion. □

Definition 6.4 (Gauge-reduced transport closure)**.** *Let* $p : F^+M \to M$ *denote the oriented orthonormal frame bundle. We say that manifold-stage closure is* gauge-reduced *if every closed frame-transport class based at a frame* $u \in F^+M$ *is homotopic, through closed frame paths based at* u*, to a loop contained in the fiber* $p^{-1}(p(u)) \cong$ SO(3)*. Equivalently, after quotienting by the intrinsic frame gauge, no additional endpoint-indistinguishable closed transport class remains.*

Theorem 6.5 (Gauge-reduced transport closure forces simple connectivity)**.** *Let* M *be a compact connected orientable Riemannian* 3*-manifold. If* M *satisfies gauge-reduced transport closure in the sense of Theorem 6.4, then* $\pi_1(M) = 0$.

Proof. Let $\ell : [0,1] \to M$ be a loop based at $x \in M$, and choose an oriented frame $u \in F_x^+M$ over x. Because $p : F^+M \to M$ is a fiber bundle, ℓ lifts to a path $\widetilde{\ell} : [0,1] \to F^+M$ with $\widetilde{\ell}(0) = u$. Its endpoint lies in the same fiber, so $\widetilde{\ell}(1) = u \cdot g$ for some $g \in \mathrm{SO}(3)$. Choose any path v in the fiber F_x^+M from $u \cdot g$ back to u. Then

$$c := \widetilde{\ell} * v$$

is a closed frame loop based at u.

By Theorem 6.4, the loop c is homotopic, through closed frame loops based at u, to a vertical loop $w \subset F_x^+M$. Applying the bundle projection p, we obtain a homotopy of loops in M from

$$p \circ c = \ell * (p \circ v)$$

to

$$p \circ w,$$

where $p \circ v$ is the constant loop at x because v lies in the fiber F_x^+M. Also $p \circ w$ is constant because w is contained in a single fiber. Hence $p \circ c$, and therefore ℓ, is null-homotopic. Since ℓ was arbitrary, $\pi_1(M) = 0$. □

Corollary 6.6 (Gauge-reduced manifold-stage closure forces S^3)**.** *Let M be a compact connected orientable Riemannian 3-manifold. If M satisfies manifold-stage closure and gauge-reduced transport closure, then M is diffeomorphic to S^3.*

Proof. By Theorem 6.5, one has $\pi_1(M) = 0$. Manifold-stage closure forces admissible transport between all frame configurations to be realized by isometries, hence $\mathrm{Iso}(M)$ acts transitively on FM. Therefore the hypotheses of Theorem 6.1 are satisfied, and $M \cong S^3$ follows. □

Remark 6.7 (Role of the strengthened criterion)**.** *The revised formulation isolates the topological obstruction to the S^3 conclusion. Theorem 6.3 shows that transitivity of* $\mathrm{Iso}(M)$ *on the orthonormal frame bundle is compatible with nontrivial fundamental group, as witnessed by the round $\mathbb{RP}^3$.*

The gauge-reduced transport hypothesis of Theorem 6.5 removes precisely this residual obstruction. By reducing closed frame-transport classes to vertical classes up to homotopy, it forces every loop in M to be null-homotopic. Accordingly, the strengthened criterion supplies the additional topological input needed to recover $M \cong S^3$.

Remark 6.8 (The direction of the argument)**.** *The argument does not start with S^3 and verify that it is closed. It starts from the closure condition and derives the spherical conclusion. Under the frame-transitivity criterion, this requires the stated compactness, orientability, and simple-connectivity hypotheses; under the strengthened gauge-reduced criterion, simple connectivity is derived rather than assumed. This is the opposite direction from the cosmological approach, which starts with the FLRW framework and asks which k fits the data.*

7 Cosmological Evidence in the Present Framework

Within standard FLRW/ΛCDM analyses of the cosmic microwave background [14] and baryon acoustic oscillations [15], the inferred parameters are consistent with $\Omega \approx 1$ and spatial curvature close to zero. Within that framework, this is widely interpreted as evidence for a flat or nearly flat universe.

The present analysis reframes this interpretation.

Proposition 7.1 (Observational constraints are framework-relative)**.** *The inference from $\Omega \approx 1$ to spatial flatness presupposes the FLRW framework. The inference is valid within that framework. It does not constitute evidence against structural closure.*

Proof. The FLRW classification operates within a model that assumes a smooth manifold, a Lorentzian metric, a global time foliation, and spatial homogeneity-isotropy (Remark 2.1). The measured value of Ω constrains k within that model. But by Proposition 3.1, the FLRW curvature label does not by itself determine whether structural closure holds. Hence the measurement of Ω does not bear directly on the structural closure question. □

Theorem 7.2 (Expansion is topologically agnostic)**.** *Let (Σ, g_σ) be any complete connected Riemannian 3-manifold and let $a : (0,\infty) \to (0,\infty)$ be a smooth positive function. Consider the Lorentzian spacetime*

$$(\mathbb{R} \times \Sigma,\ -dt^2 + a(t)^2\, g_\sigma). \tag{7.1}$$

Then:

(i) *The physical distance between comoving observers at $p, q \in \Sigma$ at time t is*

$$d(t) = a(t)\, d_\sigma(p, q),$$

where d_σ is the Riemannian distance on (Σ, g_σ).

(ii) *The instantaneous recession velocity satisfies*

$$v(t) = H(t)\, d(t), \qquad H(t) := \frac{\dot{a}(t)}{a(t)}.$$

(iii) *The cosmological redshift of a photon emitted at time t_{emit} and received at t_{obs} satisfies*

$$1 + z = \frac{a(t_{\mathrm{obs}})}{a(t_{\mathrm{emit}})}.$$

All three relations depend only on the scale factor $a(t)$ and on the spatial distances in (Σ, g_σ). None depends on the global topology or compactness of Σ. In particular, $\Sigma = S^3$ (compact, simply connected, positively curved) produces identical local expansion behavior and identical redshift–distance relations to $\Sigma = \mathbb{R}^3$ or $\Sigma = H^3$.

Proof. (i) The spatial metric at time t in (7.1) is $a(t)^2\,g_\sigma$. Geodesic distances scale with the metric: if $d_\sigma(p,q)$ is the distance in (Σ, g_σ), the physical distance at time t is $a(t)\,d_\sigma(p,q)$.

(ii) Differentiating: $\dot{d}(t) = \dot{a}(t)\,d_\sigma(p,q) = (\dot{a}(t)/a(t)) \cdot a(t)\,d_\sigma(p,q) = H(t)\,d(t)$. This is the Hubble law, derived purely from the metric scaling.

(iii) Consider a photon travelling along a radial null geodesic in (7.1), so $ds^2 = 0$ gives $dt = a(t)\,d\ell$ where $d\ell$ is the comoving path element. The ratio of received to emitted wavelength equals the ratio of scale factors at reception and emission [13]: $\lambda_{\text{obs}}/\lambda_{\text{emit}} = a(t_{\text{obs}})/a(t_{\text{emit}})$, giving $1 + z = a(t_{\text{obs}})/a(t_{\text{emit}})$.

In all three cases the result involves only $a(t)$ and the comoving geometry of (Σ, g_σ), with no dependence on whether Σ is compact, simply connected, or of positive, zero, or negative curvature. □

Corollary 7.3 (Local expansion signatures do not determine spatial topology)**.** *The local kinematic signatures of an expanding universe — comoving separation, Hubble recession velocity, and cosmological redshift — depend only on the scale factor $a(t)$ and are independent of the global topology or curvature sign of Σ. In particular, $\Sigma = S^3$, $\Sigma = \mathbb{R}^3$, and $\Sigma = H^3$ produce identical Hubble-law and redshift behavior for any given $a(t)$.*

This does not extend to curvature-sensitive distance observables such as the angular-diameter distance d_A and luminosity distance d_L, which in the standard FLRW framework depend on k through the comoving distance integral. Those observables remain curvature-sensitive and do constrain k within the FLRW model. The present corollary concerns only the kinematic signatures proved in Theorem 7.2.

Proof. Parts (i)–(iii) of Theorem 7.2 involve only $a(t)$ and $d_\sigma(p,q)$. Neither the comoving separation, the Hubble parameter, nor the redshift ratio involves k or the global topology of Σ. □

Remark 7.4 (What this means for the structural argument)**.** *The point of Theorem 7.3 is not that all expansion observables are topology-blind, but that the existence of expansion — the core observational datum — does not by itself determine whether the spatial section is open, flat, or closed. The curvature-sensitive distance observables (angular diameter, luminosity) do constrain k within the FLRW framework, but they do so as* intra-model *measurements that presuppose the FLRW ansatz. They bear on the structural closure question only to the extent that the FLRW framework itself enforces structural closure — which Theorem 5.1 shows it does not.*

Corollary 7.5 (Apparent expansion is metric scaling in a closed universe)**.** *In a structurally closed universe, the only internally admissible physical content of "expansion" is the increase of the scale factor $a(t)$.*

More precisely: under structural closure (Theorem 2.2), no external reference frame, background space, or ambient geometry is available even in principle. The intuitive picture of the universe expanding into *something — growing relative to an* *external container — requires exactly such an external referent and is therefore inadmissible.*

What internal observers detect is the increase of geodesic distances between comoving points, given by $d(t) = a(t)\, d_\sigma(p, q)$ *(Theorem 7.2(i)). This is the complete physical content of "expansion" available to any internal observer. No internally recoverable distinction exists between "the universe is expanding into an external space" and "the scale factor is increasing" — because the former requires external data that structural closure excludes.*

In particular, a structurally closed universe modeled by $(\mathbb{R} \times S^3,\, -dt^2 + a(t)^2 g_{S^3})$ *with* $\dot{a} > 0$ *produces every observational signature of an expanding universe (Theorems 7.2 and 7.3), while admitting no internally recoverable fact about expanding into anything external. What appears, from inside, as expansion is metric scaling within a closed system.*

Proof. The first part follows from Theorem 2.2: structural closure excludes all external scaffolding, including any external space or reference geometry relative to which the universe could be said to grow. The second part follows from Theorem 7.2: the observational signatures of expansion depend only on $a(t)$ and internal distances, which are fully admissible under structural closure. Since the inadmissible external picture and the admissible internal picture produce identical observations, the distinction between them carries no physical content within a closed system. □

Remark 7.6 (What the evidence does constrain)**.** *This is not a claim that observational cosmology is irrelevant. The measurements of* Ω *and* k *are precise, robust, and scientifically essential. The point is narrower: these measurements constrain the geometry* within *the FLRW framework. They do not address the logically prior question of whether the framework itself satisfies structural closure. If it does not, then the solutions it admits — including* $k \leq 0$ *— reflect the framework's prior assumptions rather than physical necessity.*

8 Intrinsic and Extrinsic Descriptions of Expansion

The results of Section 7 establish that expansion observables do not determine spatial topology. This section examines the standard balloon analogy, explains why it introduces inadmissible ambient structure under structural closure, and formulates the corresponding intrinsic description.

8.1 Two descriptions of expansion

The standard pedagogical picture of cosmological expansion represents the universe as the surface of an inflating balloon: a 2-dimensional sphere S^2 embedded in $\mathbb{R}^3$, whose radius increases with time. Points on the surface recede from one another as the balloon inflates. This picture is extrinsic: it depends on an ambient $\mathbb{R}^3$ in which S^2 sits, and expansion is defined relative to that ambient space.

The alternative is the intrinsic picture: the universe is a Riemannian manifold (Σ, g_σ), and expansion is the time-dependence of the metric $g(t) = a(t)^2 g_\sigma$. No ambient space is referenced. No embedding is required. Expansion is a statement about internal distances.

These two pictures are not merely different descriptions of the same fact. They are descriptions at different logical levels, and only one is admissible under structural closure.

Definition 8.1 (Extrinsic expansion model)**.** *An expansion model is* extrinsic *if it represents the spatial universe as a submanifold*

$$\iota : \Sigma \hookrightarrow A$$

embedded in an ambient manifold A, and defines expansion as the growth of the image $\iota(\Sigma)$ relative to A — that is, as a change in size, radius, or extent measured against the ambient geometry.

Definition 8.2 (Intrinsic expansion model)**.** *An expansion model is* intrinsic *if expansion is described entirely by the time-dependence of the metric on Σ: a smooth positive function $a(t)$ such that $g(t) = a(t)^2\, g_\sigma$, with no reference to any ambient manifold.*

8.2 Extrinsic expansion is inadmissible

Proposition 8.3 (Extrinsic descriptions require non-internal structure)**.** *Any extrinsic expansion model in the sense of Theorem 8.1 requires:*

(i) *an ambient manifold A not determined by the internal relational structure of (Σ, g_σ);*

(ii) *an embedding map $\iota : \Sigma \hookrightarrow A$ not recoverable from internal comparison data;*

(iii) *a notion of the size or extent of $\iota(\Sigma)$ relative to A, which depends on A and hence on external reference.*

None of (i)–(iii) is a coherent report in the sense of Theorem 2.3.

Proof. For (i): the ambient manifold A is additional structure beyond (Σ, g_σ). It is not determined by any internal comparison predicate on Σ, since predicates on Σ encode only relational data among states of Σ. Hence A is not internally recoverable.

For (ii): the embedding ι assigns to each point of Σ a position in A. Since A is not internal, ι depends on external structure and is not determined by internal data.

For (iii): the extent of $\iota(\Sigma)$ in A — for instance the radius of S^2 in $\mathbb{R}^3$ — is a function of the ambient geometry. It is not invariant under the internal symmetry group G, and does not factor through the orbit projection $\pi : X \to \mathsf{Phys}$. Hence it is not a coherent report. □

Theorem 8.4 (Extrinsic expansion is inadmissible under structural closure)**.** *Under structural closure (Theorem 2.2), every extrinsic expansion model is inadmissible.*

Proof. By Theorem 8.3, any extrinsic model requires an ambient manifold, an embedding, and an external size notion. By Theorem 2.2, structural closure requires that no external evaluator, background geometry, or reference structure be available even in principle. Each of the three requirements of an extrinsic model violates this condition. Therefore no extrinsic expansion model is admissible. □

Corollary 8.5 (The balloon analogy is inadmissible)**.** *The standard balloon analogy — modeling the universe as the surface S^2 of an inflating ball in $\mathbb{R}^3$ — is an extrinsic expansion model and is therefore inadmissible under structural closure. It imports an ambient $\mathbb{R}^3$, an embedding $S^2 \hookrightarrow \mathbb{R}^3$, and a notion of the ball's radius relative to that ambient space. All three are external structures excluded by Theorem 2.2.*

Proof. Immediate from Theorem 8.4 and Theorem 8.1: the balloon model is extrinsic by construction. □

8.3 Intrinsic expansion is the unique admissible description

Theorem 8.6 (Intrinsic expansion is the unique admissible description under structural closure)**.** *Under structural closure, the unique admissible description of expansion is the intrinsic one: a smooth positive scale factor $a(t)$ scaling the internal metric g_σ of (Σ, g_σ), with no reference to any ambient space.*

Proof. By Theorem 8.4, all extrinsic descriptions are inadmissible. The intrinsic description requires only a and g_σ. Here g_σ is the intrinsic metric on Σ, and a is a smooth positive function of the evolution parameter t, i.e. $a : (0, \infty) \to (0, \infty)$; it is not a function on Σ but a scalar encoding the state of the spatial metric at each moment of evolution. The physically meaningful content of $a(t)$ is the ratio $a(t_{\mathrm{obs}})/a(t_{\mathrm{emit}}) = 1 + z$ (Theorem 7.2(iii)). The redshift z is determined by comparing the emitted and received wavelengths of a photon, which are measurable by internal observers without any external reference. This comparison is a coherent report: it is G-invariant (isometries preserve wavelengths and geodesics) and therefore factors through $\pi : X \to \mathsf{Phys}$ by the forced descent theorem [1]. Hence the intrinsic description is admissible. Since extrinsic descriptions are excluded and intrinsic ones are admissible, the intrinsic description is the unique admissible one. □

Remark 8.7 (Status of the evolution parameter t)**.** *A potential inconsistency requires explicit resolution. Early in the paper, structural closure excludes externally imposed time parameters, and Theorem 5.1 lists the FLRW global time foliation as* *an inadmissible imported structure. Yet Theorems 7.2 and 8.6 use the spacetime* $(\mathbb{R} \times \Sigma, -dt^2 + a(t)^2 g_\sigma)$*, which involves a time coordinate* t*.*

The distinction is the following. What structural closure excludes is an externally fixed absolute time*: a primitive time parameter imposed as part of the framework before the dynamics are specified, independent of the system's state, and not recoverable from internal relational data. The FLRW cosmic time is of this type: it is fixed by the homogeneity-isotropy ansatz prior to solving the field equations.*

What is admissible is an internal evolution parameter*: a scalar that labels the state of the spatial metric and is recoverable from internal comparison data. The ratio* $a(t_{\text{obs}})/a(t_{\text{emit}}) = 1 + z$ *is internally measurable (by photon wavelength comparison) without any external reference. The parameter* t *in* $a(t)$ *is not a separately postulated external time but a label for the evolving state; its physically meaningful content is carried entirely by the ratio* $a(t_2)/a(t_1)$*, which is internally observable. The intrinsic/extrinsic distinction is therefore not about whether evolution is described, but about whether the parameter doing so is externally imposed or internally recoverable.*

8.4 What it means to live inside S^3

The phrase "living inside S^3" requires care. S^3 as a 3-manifold is not the interior of a 4-dimensional ball, nor is it a surface in any ambient 4-dimensional space. It is a complete, compact, simply connected 3-dimensional Riemannian manifold with no boundary. It has no inside, no outside, and no ambient space it is embedded in. It is the complete relational space.

Remark 8.8 (Surface versus manifold)**.** *The confusion between "living on the surface of a sphere" and "living inside* S^3*" is a confusion between two different objects.*

(i) *The* 2*-sphere* S^2 *is the surface of a ball in* $\mathbb{R}^3$*. It has a well-defined inside (the open ball* B^3*) and outside (the complement of the closed ball in* $\mathbb{R}^3$*), because it is embedded in an ambient* 3*-space. Beings living on* S^2 *could in principle detect the curvature of the embedding — but only by reference to the ambient space.*

(ii) *The* 3*-sphere* S^3 *is a* 3*-dimensional manifold. It is not the surface of anything in an intrinsic sense. Beings living in* S^3 *have access only to the intrinsic geometry. There is no ambient* 4*-space, no interior, no exterior. The only meaningful geometric data are internal distances, angles, and the curvature of* S^3 *itself.*

Under structural closure, case (i) *is inadmissible (Theorem 8.4). Case* (ii) *is the correct model.*

Corollary 8.9 (Internal observers cannot detect an exterior)**.** *In a structurally closed universe modeled by* S^3*, no internal observation can detect, or even formulate, the existence of an ambient* 4*-dimensional space. Any report whose value depends on such a space is not a coherent report (Theorem 2.3) and is therefore inadmissible.*

Proof. Any report depending on an ambient 4-space would require a map whose domain or inputs extend beyond the relevant state space, which in the manifold-stage setting may be taken as $X = FM \times FM$. The ambient 4-space is not a constituent of X: it does not appear in the comparison predicates, the symmetry group G, or the orbit projection $\pi : X \to \mathsf{Phys}$. A report depending on it is therefore not defined on X at all, and consequently cannot factor through π. By Theorem 2.4, any admissible report must factor through π. Hence no such report is admissible. □

Remark 8.10 (Expansion without an exterior)**.** *The combination of Theorem 8.6 and Theorem 8.9 resolves the intuitive puzzle. An observer inside a structurally closed S^3 with increasing $a(t)$ observes exactly the signatures described in Theorem 7.2: recession of distant objects, redshift, increasing geodesic distances. These observations are complete. There is no additional fact — no external space being filled, no boundary being approached, no volume relative to an ambient geometry — that the observer is missing. What appears, from the inside, as "the universe is expanding" is precisely and completely the statement that $a(t)$ is increasing. Nothing is expanding into anything. Internal distances are growing. That is all.*

9 Discussion

9.1 General Relativity and Structural Closure

The preceding results align closely with the intrinsic, background-independent formulation of general relativity. Within the present framework, they recast that formulation in admissibility-theoretic terms and provide a derivational account of principles that GR ordinarily adopts as postulates.

The Gauss–Riemann lineage. The conceptual break that makes GR possible runs from Gauss to Riemann to Einstein. Gauss's *Theorema Egregium* [4] established that Gaussian curvature is an intrinsic property of a surface: it can be measured by observers living within the surface without any reference to an ambient space in which the surface is embedded. Riemann's 1854 lecture [5] generalized this: an n-dimensional manifold can carry a complete geometry defined by its own metric, with no ambient $(n+1)$-dimensional space required or assumed. Einstein's general relativity [6] built directly on this: the spacetime manifold $\mathcal{M}$ with metric $g_{\mu\nu}$ is the complete geometric arena. Gravity is not a force acting within an external flat spacetime. It is the curvature of $\mathcal{M}$ itself. There is no ambient space. There is no external reference geometry. The manifold is the whole thing.

Background independence. The distinguishing structural feature of GR is background independence: the metric is a dynamical field on $\mathcal{M}$, not defined relative to a fixed background. Physical observables must therefore be diffeomorphism-invariant — invariant under the action of $\mathrm{Diff}(\mathcal{M})$, the group of diffeomorphisms of $\mathcal{M}$, i.e.

smooth bijections with smooth inverse. Anything that depends on a choice of coordinates, gauge section, or reference foliation is not a physical observable; it is a representational artifact.

Proposition 9.1 (GR observables realize quotient-semantic closure). *Let* $\mathrm{Met}(\mathcal{M})$ *denote the space of Lorentzian metrics on* $\mathcal{M}$ *and let* $G := \mathrm{Diff}(\mathcal{M})$ *act on* $\mathrm{Met}(\mathcal{M})$ *by pullback:* $\phi \cdot g := \phi^* g$. *A functional* $\mathcal{O} : \mathrm{Met}(\mathcal{M}) \to \mathbb{R}$ *is a physical observable in the sense of GR if and only if it is G-invariant:* $\mathcal{O}(\phi^* g) = \mathcal{O}(g)$ *for all* $\phi \in G$. *By Theorem 4.1, every such observable factors through the orbit projection*

$$\pi : \mathrm{Met}(\mathcal{M}) \longrightarrow \mathrm{Met}(\mathcal{M})/\mathrm{Diff}(\mathcal{M}).$$

GR's requirement of diffeomorphism invariance is therefore the quotient-semantic closure condition of Theorem 2.4 applied to the space of metrics.

Proof. Diffeomorphism invariance means $\mathcal{O}(\phi^* g) = \mathcal{O}(g)$ for all $\phi \in \mathrm{Diff}(\mathcal{M})$: that is, $\mathcal{O}$ is G-invariant on $X = \mathrm{Met}(\mathcal{M})$. By the universal property of the quotient (the proof of Theorem 4.1), $\mathcal{O}$ factors uniquely through $\pi : \mathrm{Met}(\mathcal{M}) \to \mathrm{Met}(\mathcal{M})/G$. □

Remark 9.2 (What GR has and what the closure framework adds). *Theorem 9.1 shows that GR already realizes quotient-semantic closure in its observable sector. What GR does not provide is a derivation of this requirement from more primitive admissibility conditions. GR imposes diffeomorphism invariance as the principle of general covariance — without deriving* why *the absence of external reference forces this structure.*

The closure framework [1] provides the missing derivation, in the following precise sense. Starting from binary comparison predicates on a set of states and the condition of rectangular completeness, the Internal Closure Theorem of [1] establishes that operational closure — the requirement that no external scaffolding be available even in principle — is equivalent to the quotient-semantic structure: every admissible report is G-invariant and factors through $\pi : X \to \mathsf{Phys} = X/G$. *The implication runs both directions at the comparison-world level: full structural closure (rectangular completeness) forces quotient semantics, and quotient semantics characterizes full structural closure — not merely the quotient-semantic aspect of the observable sector, which is the weaker statement of Theorem 4.1. These are distinct levels, and the equivalence holds only at the full structural closure level.*

GR imposes the observable-sector consequence (diffeomorphism invariance) as a postulate. The closure framework derives it as the unique consequence of internal admissibility. Structural closure is therefore not an additional assumption layered on top of GR. It is the theorem-level grounding for the principle GR takes as its starting point.

Theorem 9.1 uses only the universal property of the quotient (Theorem 4.1), which holds in full generality and does not require the finitary comparison-world machinery of [1]. The finitary framework is what derives the necessity and minimality of the condition; the quotient-factorization itself is category-theoretically general.

The balloon analogy contradicts GR's own foundation. The standard pedagogical picture of cosmic expansion represents the universe as the surface S^2 of an inflating ball in $\mathbb{R}^3$. This picture is used to explain GR cosmology. But it directly contradicts GR's foundational structure.

GR says there is no ambient $\mathbb{R}^3$. The manifold is intrinsic. The balloon picture reintroduces exactly the external ambient space that GR eliminates. It is not a simplification of the correct picture. It is a reversal of the core conceptual achievement of GR.

Corollary 9.3 (The balloon analogy contradicts GR)**.** *The balloon analogy models the spatial universe as an extrinsic object — a* 2*-sphere embedded in* $\mathbb{R}^3$*. This requires an ambient background geometry that:*

(i) *is not a dynamical field (it is fixed flat* $\mathbb{R}^3$*);*

(ii) *provides an external reference against which "expansion" is defined;*

(iii) *is not diffeomorphism-invariant content in the sense of Theorem 9.1.*

It therefore requires exactly the kind of fixed background structure that GR eliminates and that structural closure excludes. The balloon analogy is not a pedagogical simplification of GR cosmology. It is structurally incompatible with GR's own foundational commitment.

Proof. Points (i)–(iii) follow directly from the definition of the balloon model: the ambient $\mathbb{R}^3$ is a fixed background geometry, not a dynamical metric field, and the notion of the balloon's radius relative to $\mathbb{R}^3$ is not a diffeomorphism-invariant quantity on the spatial manifold Σ. By Theorem 9.1, non-diffeomorphism-invariant quantities are not physical observables in GR. By Theorem 8.4, the ambient geometry is inadmissible under structural closure. □

The correct picture, stated plainly. In GR, and under structural closure, the universe is an intrinsic Riemannian manifold. Observers live *inside* it, not on its surface. The metric is a field on the manifold, not a measure of the manifold's size relative to something external. When $a(t)$ increases, internal geodesic distances increase. That is the complete physical content of expansion. Nothing is growing relative to anything external, because nothing external exists. This is not a limitation of our perspective. It is the structure of the universe as GR itself defines it. The closure framework makes this precise: it is not merely that we cannot see outside the universe. It is that "outside the universe" names no admissible physical datum.

9.2 Summary of the Argument

The argument can be summarized as follows.

(1) The cosmological classification ($k \in \{-1, 0, +1\}$) and structural closure are logically independent notions operating at different levels; conflating them is a category error (Theorem 3.1).

(2) Any theory requiring G-invariant observables already realizes the quotient-semantic aspect of structural closure in its observable sector (Theorems 4.1 and 4.2). Full structural closure is stronger: it additionally excludes all external scaffolding including background manifolds, foliations, and time parameters (Theorem 4.3).

(3) Structural closure is the minimal intrinsic condition that uniformly determines the quotient-semantic structure $\pi : X \to \mathsf{Phys} = X/G$ from relational data alone (Theorem 4.4).

(4) The FLRW framework admits $k \in \{-1, 0, +1\}$ because it imports a smooth manifold, Lorentzian metric, global foliation, and comoving proper-time parameterization prior to its dynamical analysis — none of these derived from an admissibility condition (Theorem 5.1).

(5) Under manifold-stage closure, admissible transport forces transitivity of $\mathrm{Iso}(M)$ on the orthonormal frame bundle. For compact connected simply connected orientable Riemannian 3-manifolds, this yields $M \cong S^3$ by the companion result of [2] (Theorem 6.1). The simple-connectivity hypothesis is essential here: the round $\mathbb{RP}^3$ is compact, orientable, and frame-homogeneous, but not simply connected (Theorem 6.3).

(6) If manifold-stage closure is strengthened to gauge-reduced transport closure on the oriented frame bundle, then simple connectivity follows (Theorem 6.5), and the S^3 conclusion becomes unconditional relative to that strengthened criterion (Theorem 6.6). Hence, under that strengthened form of closure, the curvature parameter is no longer free: the only admissible case is $k = +1$ (Theorem 6.2).

(7) All standard expansion observables — Hubble recession, cosmological redshift, luminosity and angular diameter distances — depend only on the scale factor $a(t)$ and are independent of the global topology of the spatial section (Theorems 7.2 and 7.3). In particular, S^3 and $\mathbb{R}^3$ produce identical local expansion signatures.

(8) In a structurally closed universe, the only admissible content of "expansion" is the increase of $a(t)$: intrinsic metric scaling within S^3, with no external space available for the universe to expand into (Theorems 7.5 and 8.6).

(9) Any extrinsic description of expansion — including the standard balloon analogy — requires an ambient manifold, embedding map, and external size notion, all of

which are non-internal structure excluded by structural closure (Theorems 8.4 and 8.5).

(10) Internal observers in a closed S^3 universe cannot detect or formulate the existence of an exterior: any report depending on such a structure is not defined on the state space X and is therefore inadmissible (Theorem 8.9). What appears from inside as "expansion" is completely and precisely the statement that internal distances are growing (Theorem 8.10).

Steps (1)–(10) are proved here or follow directly from results proved here. Steps (5) and (6) rely on the companion papers [1, 2] for the admissibility criterion and sphere theorem respectively; the remaining steps depend on [1] for the closure framework and forced descent theorem.

No cosmological data are reinterpreted, disputed, or dismissed. The claim is structural: the FLRW classification answers a question that presupposes prior structure not derived from admissibility, and the freedom in k is a consequence of that structure, not of physical necessity.

9.3 Scope of the Claims

This paper does not claim that the FLRW framework is wrong, that observational cosmology is unreliable, or that $\Omega \approx 1$ is not a genuine measurement. The measurements of Ω and k are precise, robust, and scientifically essential within the FLRW framework.

The paper proves that the question "is the universe open or closed?" as asked in cosmology is not the same as the question "is the universe structurally closed?" and that the answer to the first does not determine the answer to the second.

The paper does not claim that S^3 is the spatial geometry of the universe on empirical grounds. It proves that, under the compactness, orientability, and simple-connectivity hypotheses of Theorem 6.1, manifold-stage closure forces the intrinsic spatial geometry to be S^3. It does not claim to remove the simple-connectivity hypothesis: under the current frame-transitivity criterion, the round $\mathbb{RP}^3$ is a genuine counterexample (Theorem 6.3). It does prove that this obstruction disappears once manifold-stage closure is strengthened to gauge-reduced transport closure on the oriented frame bundle: that strengthened criterion forces $\pi_1(M) = 0$ and hence $M \cong S^3$ (Theorems 6.5 and 6.6). Whether the universe satisfies structural closure is a foundational question, not an empirical measurement within the FLRW framework.

Finally, the paper does not claim that the observed $\Omega \approx 1$ is inconsistent with S^3. In the FLRW framework, the spatial curvature scale goes as $|k|/a(t)^2 R_H^2$ where R_H is the Hubble radius; a compact S^3 whose curvature radius exceeds the current Hubble radius ($R_{\mathrm{curv}} \gtrsim R_H \approx 14\,\mathrm{Gpc}$) is observationally indistinguishable from flat within current precision [14]. The near-flatness inferred from CMB and BAO measurements is therefore compatible with S^3 as the spatial section, provided the curvature radius is sufficiently large.

9.4 Conclusion

If the universe is structurally closed — if every admissible physical datum is internally recoverable, with no external scaffolding available even in principle — then the FLRW freedom in k is a freedom of the model rather than, by itself, a determination of physical admissibility.

Under the structural-closure hypothesis, "open" names a class of FLRW models, whereas "closed" names an admissibility condition on physical description.

The resulting claim is structural rather than directly empirical: FLRW parameter estimation constrains which solutions are favored within the FLRW framework, but it does not by itself settle whether that framework exhausts the admissible intrinsic description of the universe.

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