Foundational Errors in Physical Theories:
A Structural Taxonomy

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2020 Mathematics Subject Classification. 83-02, 81-02, 83C45, 18B40 (primary); 53C29, 81P05, 81R05 (secondary).

Keywords. closed systems; external observer; measurement problem; problem of time; gauge symmetry; subsystem attribution; irreversibility; foundational errors; relational mechanics.

Abstract

This paper introduces a formal class of structural framework data and four associated axes of structural input: externalization, artificial factorization, premature globalization, and reification of repair. A theorem establishes their logical independence, so no axis is a Boolean consequence of the other three. The result yields an irreducible four-axis basis for a structural taxonomy of foundational error. The paper then applies this taxonomy to recurring tensions in classical mechanics, quantum mechanics, quantum field theory, general relativity, and statistical mechanics. Within this framework, the measurement problem, the problem of time, the status of gauge symmetry, the interpretation of renormalization, and the origin of irreversibility are treated as arising from different combinations of the same independent structural moves. No claim of resolution is made. The aim is to define the framework, prove the independence theorem, and formulate interpretive applications and open problems.

1 Introduction

Several canonical foundational problems in physics remain unresolved and are often studied in isolation. The measurement problem in quantum mechanics has resisted resolution since the Copenhagen era [29, 47]. The problem of time in quantum gravity has remained open since its canonical formulation by DeWitt [15] and subsequent analyses [31, 34, 6]. The status of gauge symmetry, the origin of irreversibility, and the interpretation of renormalization have likewise generated enduring foundational debates, reflected in classic work on gauge structure, irreversibility, and renormalization [44, 54, 12, 36, 51].

Main theorem. This paper introduces a class of structural framework data with four associated axes of structural input: externalization, artificial factorization, premature globalization, and reification of repair. Theorem 3.1 establishes their logical independence: no axis is a Boolean consequence of the other three. This yields a minimal four-axis basis for the taxonomy developed below.

Interpretive scope. The remainder of the manuscript applies that formal result to standard foundational episodes. It does not claim theorem-level reductions of each historical problem to a unique mathematical obstruction. Rather, it argues that many familiar tensions can be organized by combinations of the four axes once closed-world admissibility is taken as the organizing criterion. On this view, the deepest failures lie not in local defects of otherwise correct equations but in structural inputs fixed prior to dynamics: primitive evaluation data, subsystem cuts, imported global carrier structure, and the ontological treatment of repair devices.

Relation to earlier literature. Elements of this structure appear separately in earlier work. Mach emphasized the problem of absolute space [37]. Haag identified a structural obstruction in quantum field theory [27]. Singer identified a global obstruction in gauge theory [44]. Bell identified an obstruction to local hidden-variable completion of EPR-type correlations [10]. DeWitt exhibited the disappearance of an external time parameter in canonical quantum gravity [15]. The present paper places these pressures within a single formal taxonomy and establishes the logical independence of its four axes.

The paper proceeds as follows. Section 2 introduces the formal framework and its four axes. Section 3 proves Theorem 3.1. Section 4 records the taxonomic assignments. Section 5 relates standard foundational problems to combinations of the four errors. Section 6 states the resulting open problems. Section 7 records the compressed core statement. Section 8 distinguishes the theorem-level result from its interpretive applications.

2 Formal Setup

We work with four structural predicates on framework data:

I. Externalization. Primitive evaluation, state, arena, time, or measure is placed outside the system being described.

II. Artificial factorization. A subsystem decomposition is imposed as primitive rather than derived from the relational structure of the universe.

III. Premature globalization. Global carrier structure is imposed before compatibility with the relational constraints has been verified.

IV. Reification of repair. Compensating structures introduced to repair an obstruction are then treated as ontology rather than bookkeeping.

The following definitions fix the formal setting.

Definition 2.1 (Closed universe)**.** *A* closed universe *is a world U in which every evaluator, clock, reference frame, and measuring device is an internal constituent of U.*

Definition 2.2 (Closed-world admissibility)**.** *A quantity Q defined on U is* closed-world admissible *if (i) Q is invariant under all internal symmetries of U, (ii) Q is independent of cut choices used to partition U into subsystems, and (iii) Q is independent of representational choices — coordinates, gauge sections, foliations — that are not themselves dynamical degrees of freedom of U.*

Definition 2.3 (Externalization)**.** *A framework* externalizes *if it postulates a primitive evaluation map*

$$\mathrm{Eval} : S \times O \to V,$$

whose values are not determined by relations among internal constituents of U.

Definition 2.4 (Artificial factorization)**.** *A framework* artificially factorizes *if it posits a primitive decomposition $U = A \otimes B$ (or $U = A \cup B$) not derived from the relational structure of U.*

Definition 2.5 (Premature globalization)**.** *A framework* prematurely globalizes *if it assigns global structure — for example, a Hilbert space, smooth manifold, probability measure, or global time parameter — before proving that the relational constraints of U are compatible with that structure.*

Definition 2.6 (Reification of repair)**.** *A framework* reifies repair *if, after an obstruction is encountered, it introduces a compensating structure and then treats that structure as primitive ontology rather than bookkeeping.*

Definition 2.7 (Structural framework datum)**.** *A* structural framework datum *is a tuple*

$$\mathfrak{F} = (U, \mathrm{Eval}, \Pi, \Gamma, \mathcal{R}),$$

where U is a closed universe, Eval *is either absent or an evaluation map as in Definition 2.3,* Π *is either absent or a primitive subsystem decomposition,* Γ *is either absent or a package of global carrier structure, and* $\mathcal{R}$ *is a family of compensating structures.*

Definition 2.8 (Axis predicates on framework data)**.** *For a structural framework datum $\mathfrak{F}$, write $I(\mathfrak{F})$, $II(\mathfrak{F})$, $III(\mathfrak{F})$, and $IV(\mathfrak{F})$ for the satisfaction of Definitions 2.3, 2.4, 2.5, and 2.6 respectively. In particular, $IV(\mathfrak{F})$ requires that some element of $\mathcal{R}$ be promoted to primitive ontology rather than retained as bookkeeping.*

3 Independence Theorem

Theorem 3.1 (Logical independence of the four axes)**.** *On the class of structural framework data, the predicates* I*,* II*,* III*, and* IV *are logically independent. Equivalently, for each* $X \in \{I, II, III, IV\}$ *there exists a structural framework datum* $\mathfrak{F}_{\neg X}$ *such that* $X(\mathfrak{F}_{\neg X})$ *fails and the remaining three predicates hold.*

Proof. Fix a closed universe U, a primitive evaluation map Eval, a primitive subsystem decomposition Π, a package Γ of global structure, and nonempty families $\mathcal{R}_{\text{prim}}$ and $\mathcal{R}_{\text{book}}$ of compensating structures, the former designated primitive and the latter bookkeeping only.

Define the witness data

$$\mathfrak{F}_{\neg I} = (U, \varnothing, \Pi, \Gamma, \mathcal{R}_{\text{prim}}), \qquad \mathfrak{F}_{\neg II} = (U, \text{Eval}, \varnothing, \Gamma, \mathcal{R}_{\text{prim}}),$$

$$\mathfrak{F}_{\neg III} = (U, \text{Eval}, \Pi, \varnothing, \mathcal{R}_{\text{prim}}), \qquad \mathfrak{F}_{\neg IV} = (U, \text{Eval}, \Pi, \Gamma, \mathcal{R}_{\text{book}}).$$

By inspection of the defining components, $\mathfrak{F}_{\neg I}$ lacks only the evaluation map, so $I(\mathfrak{F}_{\neg I})$ fails while $II(\mathfrak{F}_{\neg I})$, $III(\mathfrak{F}_{\neg I})$, and $IV(\mathfrak{F}_{\neg I})$ hold. Likewise, $\mathfrak{F}_{\neg II}$ lacks only the primitive decomposition, $\mathfrak{F}_{\neg III}$ lacks only the global carrier structure, and $\mathfrak{F}_{\neg IV}$ differs only in that its compensating structures are retained as bookkeeping; hence $II(\mathfrak{F}_{\neg II})$, $III(\mathfrak{F}_{\neg III})$, and $IV(\mathfrak{F}_{\neg IV})$ fail respectively, while the remaining three predicates hold in each case.

Thus every axis fails on some datum on which the other three hold. Therefore no axis is a Boolean consequence of the remaining three, and the four predicates are logically independent. □

Corollary 3.2. *Any identification of two distinct axes loses information: some structural framework data distinguished by the full four-axis taxonomy become indistinguishable after the identification.*

4 Taxonomic Assignments

This section assigns representative foundational episodes to the four axes. For each item, it states the structural content of the assignment and cites literature in which the same pressure appears.

4.1 Axis I: Externalization

Axis I collects failures in which evaluative, kinematical, or representational data are treated as externally fixed.

I.1 External evaluator postulate. Error I.1 consists in representing measurement by an external evaluation map $\text{Eval}(\text{state}, \text{setting}) = \text{outcome}$. In a closed universe, evaluation must be represented by relations among internal loci.

This error is the foundational commitment of the Copenhagen interpretation. Bohr treated the experimental arrangement as classically described rather than as part of a single quantum-state description of the system [11], and Heisenberg treated the cut between observer and observed as unavoidable though not fixed once and for all by the formalism [29]. Von Neumann's projection postulate provides the standard formal expression for an external measurement update [47]. Wigner's friend thought experiment [50] and its modern extensions [23] sharpen the point: when agents model one another inside a single closed quantum description, additional single-world consistency assumptions can become mutually incompatible. The measurement problem in every formulation — from Everett [21] to decoherence theory [57] — is the unresolved consequence of Error I.1.

I.2 Global state as absolute object. Physics assumes the state exists as an absolute element of reality: $x \in X$, $|\psi\rangle \in \mathcal{H}$, $\phi \in \mathcal{F}$, $g_{\mu\nu} \in \mathcal{G}$. A closed world has no canonical state independent of internal relational context.

Einstein's EPR paper [20] introduced a criterion of physical reality framed in terms of predictability without disturbance. In the present taxonomy, that criterion is read as pressure toward an absolute state assignment independent of the measuring procedure. Bell's theorem [10] demonstrated that no local hidden-variable theory can reproduce all of the EPR-type correlations predicted by quantum mechanics.

I.3 Hidden god-frame via coordinate or gauge choices. Physics treats coordinate and gauge choices as harmless conventions, then interprets results as absolute features of U. In a closed universe, conventions are internal structure; results that depend on them are not closed-world admissible.

Mach's critique of Newton's absolute space [37] identified this error in classical mechanics. Kretschmann showed in 1917 that general covariance does not eliminate background structure [33]: any theory can be written in generally covariant form. Anderson's absolute objects formalism [5] was a systematic attempt to identify which structures in general relativity remain external anchors despite formal covariance. Norton's analysis of the hole argument [39] sharpened the distinction between general covariance, manifold point identification, and genuine background independence. The resulting lesson is not that standard general relativity contains a fixed background metric, but that covariance by itself does not remove all surplus representational structure.

I.4 External time parameter as primitive. Physics postulates t as a primitive and writes dynamics as ∂_t laws, Hamiltonian flow. Global time is an externalization.

Dirac's Hamiltonian analysis of constrained systems [18] supplied the general framework. In canonical general relativity, the ADM formulation [2] makes the total Hamiltonian a sum of constraints, so that on shell one has $H_{\text{total}} \approx 0$. There is no external time to generate dynamics. DeWitt [15] derived the Wheeler-DeWitt equation $H\Psi = 0$ by quantizing this constraint, producing a timeless equation whose interpretation has remained contested since. Isham's comprehensive review [31] catalogs the major strategies for recovering time internally and the conceptual difficulties they face. Barbour's program [7] is a sustained argument that time does not exist as an external primitive. Anderson's monograph [6] documents the breadth of the problem. Error I.4 names one of the most widely acknowledged foundational problems in quantum gravity; no agreed resolution exists.

4.2 Axis II: Artificial factorization errors

II.1 Subsystem split treated as natural. Physics assumes an objective decomposition $A|B$ as primitive input. A subsystem split is extra structure; observables produced from a split are conditional on the cut and hence not closed-world admissible.

Zanardi showed that the decomposition of a quantum system into subsystems is basis-dependent: there is no canonical tensor product structure in a generic Hilbert space [55]. In algebraic quantum field theory, the primary objects are local observable algebras attached to spacetime regions rather than a preferred global tensor-factor split into "system" and "environment" [28]. Haag's local observable algebras [28] were motivated precisely by the need to work with local regions without imposing a global factorization.

II.2 Environment freezing produces artificial absolutes. Physics replaces the rest of U by fixed boundary conditions, fixed clocks, fixed reference frames, fixed vacua. Results are conditional on the frozen environment, hence not fundamental.

Caldeira and Leggett's influential model of quantum Brownian motion [13] describes a system coupled to an explicit dissipative environment, which in the present taxonomy is an environment freeze. The vacuum state of quantum field theory is similarly a fixed external anchor: changes of vacuum (Bogoliubov transformations, Unruh effect [46]) can yield observer-dependent particle descriptions — a manifestation of Error II.2.

II.3 Pairwise reduction of multi-point structure. Physics reduces interaction to binary primitives. Multi-way compatibility constraints are not determined by any single pair in isolation; the first nontrivial invariants are loop or triangle constraints.

Mermin's GHZ argument [24, 38] exhibited genuinely multipartite constraints that are not visible in any single pair taken in isolation. Abramsky and Brandenburger's sheaf-theoretic analysis of contextuality [3] proved that even overlap-compatible local assignments need not extend to a single global assignment.

II.4 Open system elevated to ontology. Physics elevates open-system dynamics — noise, decoherence channels, Lindblad operators — to fundamental descriptions of reality. Openness models ignorance about the rest of a closed universe; it cannot be primitive without positing an external bath.

The Lindblad equation [35] characterizes Markovian completely positive semigroup dynamics for an open subsystem. Treating it as a fundamental equation of motion imports an effective environment or coarse-graining assumption, which is the present instance of Error II.4.

4.3 Axis III: Premature globalization errors

III.1 Background carrier space assumed. Physics starts with a manifold M as a given carrier. In a closed relational world, "where" cannot be primitive.

Mach [37] argued against Newton's absolute space. Earman and Norton [19] showed through the hole argument that manifold point identification can carry surplus representational structure not fixed by observable fields. Background independence is treated in the quantum-gravity literature as a central unfinished issue [45].

III.2 Global Hilbert space as primitive. Physics assumes a single global Hilbert space with linear superposition.

Haag's theorem [27] shows that the interaction-picture idealization of quantum field theory — which requires a unitary equivalence between free and interacting representations — fails in the standard infinite-volume setting. That theorem does not by itself settle the status of renormalized quantum field theory, but it does show that the naive global-Hilbert- space picture has nontrivial mathematical consequences. Perturbative renormalization can then be read as a repair strategy for working around this failure, rather than as a direct theorem-level consequence of Haag's result. Haag's algebraic quantum field theory program [28] is a sustained attempt to formulate the theory in terms of local algebras and their representations rather than a single preferred global interaction-picture Hilbert-space representation.

III.3 Global time foliation treated as given. General relativity has no preferred foliation of spacetime.

In canonical gravity, this appears as the absence of any preferred decomposition into spatial slices plus an external time parameter [2, 34, 31]. The ADM formalism [2] introduces lapse and shift functions as compensators for the absence of a preferred foliation — these are repair structures for the premature assumption of Error III.3. Kuchař's analysis [34] identifies the choice of internal time variable as the central underdetermined freedom in canonical quantum gravity.

III.4 Infinitesimalization before integrability. Infinitesimal gauge-fixing does not extend globally.

Singer's theorem [44] gives the mathematical form of this obstruction: for non-abelian Yang-Mills theory on compact Euclidean spacetime, no gauge condition defines a global section of the gauge-orbit space. The Gribov ambiguity [26] is the corresponding physical manifestation: standard gauge conditions can intersect a single gauge orbit more than once, so the Faddeev-Popov procedure fails globally. This is the structural consequence of Error III.4: the infinitesimal gauge condition is locally consistent but globally obstructed. Singer formulated the issue as a global geometric obstruction in the space of gauge potentials [44].

III.5 Probability as primitive measure. The Born rule is postulated as a primitive.

Gleason's theorem [25] characterizes additive probability measures on the closed subspaces of a Hilbert space, thereby underwriting the Born-rule form on projectors, but it does not derive the Hilbert space itself. Everett [21] attempted to derive probability from the relative state formulation without postulating it. Deutsch [16] and Wallace [48] attempted derivations from decision theory, each acknowledging that probability is not a primitive. Frauchiger and Renner [23] sharpened the measurement-theoretic tension in nested-observer scenarios under additional consistency assumptions, but did not supply a derivation of probability from internal structure.

III.6 Global smooth structure at bedrock. Differentiability and PDE structure are assumed at the foundational level.

Penrose's twistor program [40] recasts spacetime structure in terms of twistor space and complex-geometric data rather than ordinary spacetime points. Regge calculus [42] discretizes spacetime to avoid importing smooth structure as primitive. Connes's noncommutative geometry [14] replaces the smooth manifold with a spectral triple derived from algebraic data. Each of these programs is a response to Error III.6 without agreement on a resolution.

4.4 Axis IV: Reification of repair errors

IV.1 Gauge data reified as dynamical substances. Physics introduces gauge redundancy to patch comparison consistency, then risks treating gauge-dependent connection data, gauge-invariant field strength, and loop or holonomy observables as if they were the same kind of primitive entity.

The Aharonov-Bohm effect [4] showed that connection data can have observable consequences even when the local field strength vanishes on the relevant region, forcing a distinction between gauge-dependent potential, gauge-invariant field strength, and global holonomy data. Wu and Yang [54] identified holonomy — the path-ordered exponential around a closed loop — as a gauge- invariant carrier of physical information. Error IV.1 is not the use of these distinct objects, but the failure to keep straight which are gauge-dependent bookkeeping devices and which encode invariant content.

IV.2 Curvature reified as a fundamental field. The metric is often treated as primitive geometric data, while curvature is then granted independent ontological weight beyond its status as structure derived from the metric or connection.

Weyl's original unified theory [49] treated electromagnetism as the curvature of an extended geometry, reifying the gauge repair structure. The Kobayashi-Nomizu formulation [32] of connections on principal bundles is a partial resolution: it recognizes curvature as transport data on a bundle. But the bundle itself is still imposed as external background, not derived from internal comparison structure.

IV.3 Global gauge-fixing assumed possible. Physics assumes a global gauge section can always be chosen cleanly.

Singer's theorem [44] and the Gribov ambiguity [26] show that this assumption fails in the standard non-abelian Yang-Mills settings where Gribov copies occur. The standard perturbative treatment uses Faddeev-Popov ghosts [22] to enforce gauge-fixing locally, introducing further repair structures — ghost fields — for the failure of the global section. Each layer of repair (BRST symmetry [9], Batalin-Vilkovisky formalism [8]) is a further reification of a compensating structure.

IV.4 Irreversibility as fundamental. Entropy production is treated as ontic rather than as coarse-grained bookkeeping.

Boltzmann's H-theorem [12] derived irreversibility from reversible Newtonian mechanics using the Stosszahlansatz (molecular chaos assumption). Loschmidt's reversibility objection [36] and Zermelo's recurrence objection [56] showed immediately that the derivation requires an assumption not derivable from the reversible base theory. Boltzmann acknowledged this and retreated to statistical arguments. Prigogine's program [41] attempted for decades to derive irreversibility from reversible dynamics. These difficulties do not prove that openness is the only available source of time-asymmetric behavior, but they do show that standard derivations require extra assumptions — coarse-graining, typicality, special initial data, or openness — beyond reversible microdynamics itself. Error IV.4 arises when one such repair is then reified as fundamental.

IV.5 Anomalies treated as ad hoc constraints. Anomalies appear; physics imposes cancellation conditions as unexplained requirements.

The Adler-Bell-Jackiw anomaly [1] showed that the classical axial current conservation is broken by quantization. The standard response imposes anomaly cancellation as a consistency condition on the matter content. Witten's global $SU(2)$ anomaly [53] showed that an odd number of Weyl fermion doublets makes the fermion determinant change sign under a large gauge transformation, forcing an explicit restriction on admissible matter content. Both are treated as ad hoc restrictions rather than as structural consequences of earlier errors.

IV.6 Renormalization group reified as running physics. Wilson's renormalization group [51] provides a powerful framework for relating effective descriptions across scales. The identification of the renormalization group flow with "running couplings" that are independently physically real is Error IV.6. The renormalization group is a map between effective descriptions at different resolution scales. The divergences it regulates arise from premature globalization (Error III.6): the imposition of a smooth continuum before verifying that the relational structure is compatible with it everywhere.

5 Consequences: The Foundational Problems as Error Instances

This section assigns canonical foundational problems to combinations of the four errors. The claims below are structural assignments rather than theorem-level reductions.

Measurement problem. The measurement problem is assigned to Error I.1 (Bohr [11], von Neumann [47], Everett [21], Wigner [50]). On this assignment, the observer-system cut is not a feature of the physical world but an artifact of the external evaluator postulate. Collapse, many worlds, decoherence, and relational quantum mechanics [43] are then read as attempts to eliminate or relocate the external evaluator without a structural account of what replaces it.

Problem of time. The problem of time is assigned to the conjunction of Errors I.4 and III.3 (Dirac [18], DeWitt [15], Isham [31], Kuchař [34], Anderson [6]). The Wheeler-DeWitt equation $H\Psi = 0$ contains no time variable because time was postulated externally and general relativity does not preserve that postulate under quantization. Attempts to recover time internally from the equation have generated multiple competing strategies rather than a unique construction [31, 6].

Status of gauge symmetry. The status of gauge symmetry is assigned to the conjunction of Errors IV.1 and III.4 (Dirac [18], Henneaux-Teitelboim [30], Singer [44], Wu-Yang [54]). Gauge comparison data enter as repair terms for local comparison inconsistencies and are then liable to reification unless connection, field strength, and holonomy are kept sharply distinct. Singer's theorem shows that no global gauge section need exist. Wu and Yang show that holonomy carries gauge-invariant information, whereas the connection itself is gauge-dependent; the reification pressure is therefore already present in the formalism.

Origin of irreversibility. The origin of irreversibility is assigned to the conjunction of Errors IV.4 and II.2 (Boltzmann [12], Loschmidt [36], Prigogine [41]). The base dynamics of classical and quantum mechanics is time- symmetric. Standard derivations of irreversibility require additional asymmetry-bearing assumptions, often in the form of environment freezing (Error II.2), coarse-graining, or special initial data. Entropy production is then liable to be treated as fundamental (Error IV.4). Loschmidt's objection shows that irreversibility does not follow from time-symmetric dynamics without such further structure.

Charge quantization. Charge quantization is assigned here to the conjunction of Errors III.1 and III.6. Standard derivations proceed through topological arguments about the global bundle structure of gauge fields or through Dirac's monopole argument [17, 54]. On this assignment, the topology that quantizes charge is not derived purely from local relational data but enters through imported global bundle structure together with global smoothness.

Anomalies. Anomalies are assigned to the conjunction of Errors IV.1, III.4, and IV.5 (Adler-Bell-Jackiw [1], Witten [53]). Gauge symmetry is introduced to repair comparison consistency (Error IV.1), globally gauge-fixed by a procedure that fails non-perturbatively (Error III.4), and the resulting topological obstructions are then treated as unexplained cancellation constraints (Error IV.5).

6 The Errors as Open Problems

Each error in the taxonomy generates a corresponding open problem. We state them precisely.

Problem I.1. Give a mathematically precise formulation of measurement in a closed universe that does not postulate an external evaluator, an external state space, or an observer-system cut not derived from the internal structure of the universe.

Problem I.2. Characterize physical states in a closed universe without reference to an absolute state space. What replaces the Hilbert space vector when the observer is internal?

Problem I.3. Give a criterion for closed-world admissibility of a physical quantity that is independent of coordinate choice, gauge section, and foliation. Characterize which quantities in general relativity and gauge theory satisfy this criterion.

Problem I.4. Derive the temporal succession of events in a closed universe from internal relational data, without postulating a global time parameter. What is the precise theorem-level condition on internal structure that is necessary and sufficient for a global time parameter to exist?

Problem II.1. Characterize the admissible decompositions of a closed universe into subsystems in a way that does not depend on an externally imposed cut. What physical invariants, if any, survive all possible cut choices?

Problem II.3. Identify the minimal arity at which genuinely new multi-point constraints appear that are not determined by pairwise data. Give a systematic classification of such constraints.

Problem III.2. Derive the Hilbert space structure of quantum mechanics from more primitive relational data, without postulating a global vector space. What internal structure of a closed relational system forces linear superposition?

Problem III.4. Give a complete classification of the obstruction to global gauge-fixing for non-abelian gauge theories. Singer's theorem [44] establishes existence of the obstruction. The present paper does not attempt a complete classification of its structure.

Problem III.5. Derive the Born rule from internal indistinguishability or symmetry data of a closed system, without postulating a probability measure.

Problem IV.1. Give a formulation of gauge theory in which the gauge potential does not appear as a primitive variable. Characterize the invariant content of gauge theories entirely in terms of holonomy data, without reference to a chosen gauge section.

Problem IV.2. Derive the metric and curvature of spacetime from more primitive relational or combinatorial data, without postulating a smooth manifold as background.

Problem IV.4. Derive the arrow of time from the internal dynamics of a closed reversible system, without postulating an asymmetric environment or an entropy primitive. Give a precise theorem-level statement of the conditions under which irreversibility appears as a derived rather than fundamental feature.

No separate problem statements are given for Errors II.2, II.4, III.1, III.3, III.6, IV.3, IV.5, and IV.6. In the present organization, II.2 and II.4 are treated as subsystem-environment specializations of Problem II.1; III.1 and III.6 as geometric versions of Problem IV.2; III.3 as a temporal specialization of Problem I.4; IV.3 as the global form of Problem III.4; IV.5 as downstream from Problems IV.1 and III.4; and IV.6 as downstream from Problems III.2 and IV.2. The list above therefore records only the non-redundant open problem statements.

7 The Compressed Core

This section records a one-sentence compression of the four-axis taxonomy.

> **Physical frameworks externalize, factorize, and globalize before verifying relational integrability; they then reify the compensators used to repair the resulting obstructions.**

This statement is schematic rather than exhaustive. Each item in Section 4 is a domain-specific instance of the same compression. The assignments in Section 5 accordingly treat standard foundational problems as different combinations of the same four independent errors.

8 Conclusion

This paper establishes a formal taxonomy of structural input for foundational physical theories. Definitions 2.3, 2.4, 2.5, and 2.6 specify four predicates — externalization, artificial factorization, premature globalization, and reification of repair — on structural framework data. The precise formal result is Theorem 3.1: these predicates are logically independent on the class introduced in Section 2.

The remainder of the paper is interpretive: it assigns foundational episodes to combinations of the four independent errors, records representative literature, and formulates the associated reconstruction problems. No claim is made that each historical episode is thereby reduced to a unique formal obstruction or that any listed problem has been resolved.

The broader conclusion is modest: recurring tensions often treated as separate can be organized by four kinds of input fixed prior to dynamics: primitive evaluation data, imposed subsystem cuts, imported global carrier structure, and the ontological promotion of repair devices. Until the open problems in Section 6 are resolved, foundational physical theory, on the present analysis, continues to rely on structural inputs whose closed-world admissibility has not yet been established. For a companion development of the closed-system program for Errors I, II, and IV, see Wolfe's *Diagonal Redundancy and the Obstruction to Subsystem Attribution* [52].

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