Foundational Closure and Primitive Structural Input: A Four-Axis Taxonomy

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Abstract

This paper develops a conditional structural taxonomy for frameworks that present themselves as foundational accounts of a closed world. The organizing criterion is closed-world admissibility: relative to such an account, a quantity or structure should be invariant under internal symmetries, independent of inadmissible subsystem cuts, and independent of non-dynamical representational choices. On that basis, the paper isolates four recurrent kinds of primitive structural input: externalization, artificial factorization, premature globalization, and reification of repair. It formalizes these inputs as predicates on structural framework data and proves that they are logically independent on that class: no axis is a Boolean consequence of the other three. The resulting four-axis scheme offers a way to classify recurring tensions in classical mechanics, quantum mechanics, quantum field theory, general relativity, and statistical mechanics. The classification is a proposed structural diagnosis rather than a claimed solution of the measurement problem, the problem of time, gauge reduction, renormalization, or irreversibility. Its purpose is to identify the reconstruction burden that arises, within the closure program, when a successful local formalism relies on structural input whose closed-world admissibility remains unestablished.

1 Introduction

Several canonical foundational problems in physics remain unresolved and are often studied separately. The measurement problem in quantum mechanics is rooted in the

Copenhagen-era separation of system and measurement [29, 47]. The problem of time in quantum gravity has remained open since its canonical formulation by DeWitt [15] and subsequent analyses [31, 34, 6]. The status of gauge symmetry, the origin of irreversibility, and the interpretation of renormalization have likewise generated enduring foundational debates, reflected in classic work on gauge structure, irreversibility, and renormalization [44, 54, 12, 36, 51]. The organizing question is therefore conditional and explicit: if the universe is treated as a closed system, what structural input may still be used without being derived from within that system? Readers who reject the closed-system premise need not accept the taxonomy as a normative demand on all foundational physics; their disagreement lies at the methodological premise from which the reconstruction program begins.

This premise is not derived from physics alone and is not forced on every legitimate use of physical theory. Effective realism, pragmatic modeling, and pluralist accounts may allow external clocks, subsystem cuts, background structures, or repair devices to remain primitive within limited domains. The claim of this paper is narrower and programmatic: when a framework is given universal scope and presented as a foundational account of the closed world, those same inputs become reconstruction burdens unless they are derived internally, shown to be eliminable, or acknowledged as limits of the framework's closed-world ambition.

Contribution. The contribution has three parts. First, the paper formulates closed-world admissibility as a criterion for assessing primitive structural input within the closure program. Second, it isolates four recurrent roles for such input: external evaluation data, primitive subsystem cuts, imported global carrier structure, and compensating structures promoted from repair devices to ontology. Third, it turns these four inputs into conditional reconstruction problems: an internal evaluator, an admissible subsystem decomposition, compatible global carrier structure, or a non-ontological account of repair devices must be supplied rather than assumed.

Main theorem. The formal result is a logical independence theorem for the four axis predicates on structural framework data. Theorem 5.1 shows that no axis is a Boolean consequence of the other three on the stipulated class. This result is not a dynamical independence theorem inside a particular physical theory; it shows that the four structural inputs cannot be collapsed into fewer Boolean axes without losing distinctions already present in the framework data.

Why these four axes. Given the closure criterion, the four axes arise from the ways a closed-system description can depend on structure not yet accounted for inside the system. A closed-system description can fail admissibility by placing *evaluation* outside the system, by imposing a *subsystem cut*, by importing *global carrier structure* before compatibility has been established, or by assigning primitive status to a *compensating repair* after an obstruction has appeared. The first three roles concern direct primitive input; the fourth concerns compensatory primitive input. Both count as primitive structural input once they are made explanatory or ontological in the framework data used in the taxonomy. The exhaustiveness claim is correspondingly bounded and formalized in Proposition 4.13: relative to the framework data considered here, primitive input can enter as an external evaluator, an imposed cut, imported global carrier structure, or an ontologized compensator. A proposed fifth axis would therefore have to identify another structural role not reducible to these four within the same class of data. The independence theorem records that these roles are logically separable within that framework class.

Scope. The formal result is applied to standard foundational episodes without claiming to solve the measurement problem, the problem of time, gauge reduction, renormalization, or irreversibility one by one. The classifications are structural diagnoses: they identify which primitive input is doing explanatory work relative to closed-world admissibility. They are not claimed to be unique historical readings of the cited literatures, nor derivations of the only possible interpretation of the underlying physics. The target is therefore not a defect in otherwise successful equations but the prior structural input on which those equations are made to depend.

Use of the word "error." The word "error" is used as a technical term for a mismatch between primitive structural input and the closed-world admissibility criterion. It is not used in a historical, methodological, or sociological sense. It does not mean that the theories under discussion are empirically unsuccessful or that the cited authors committed methodological errors. Such inputs may be effective idealizations in ordinary practice; they count as foundational errors, in the technical sense used here, only when they are treated as primitive, explanatory, or ontological in a closed-world account.

Relation to earlier literature. Elements of this structure appear separately in earlier work. Mach emphasized the problem of absolute space [37]. Haag identified a structural obstruction in quantum field theory [27]. Singer identified a global obstruction in gauge theory [44]. Bell identified an obstruction to local hidden-variable completion of EPR-type correlations [10]. DeWitt exhibited the disappearance of an external time parameter in canonical quantum gravity [15]. This paper places these pressures within a single formal taxonomy and establishes the logical independence of its four axes on the stipulated class of framework data.

The resulting structure consists of a formal framework, an independence theorem, an axis-by-axis taxonomy, assignments of standard foundational problems, and the reconstruction criteria implied by those assignments.

2 Why Foundational Closure?

The closure requirement is not introduced as a metaphysical axiom. It is an explanatory standard for theories that claim universal scope. Local theories may borrow clocks, boundary conditions, environments, apparatuses, or asymptotic regions from their modeling context without inconsistency. A universal-scope theory is different: if the target is the closed world, there is no exterior from which such resources can be borrowed without becoming part of what is to be explained. The Closure Program makes that added burden explicit: derive the resources internally, show that they are eliminable bookkeeping, or state where closed-world scope ends.

The motivation is also historical. Foundational physics repeatedly introduces structures whose status is later problematized: observers in measurement theory, absolute or external time in dynamics, subsystem cuts in quantum theory, backgrounds and foliations in gravitation, gauge choices in field theory, coarse-grainings in statistical mechanics, and renormalization or ghost structures in quantum field theory. Each may be indispensable in successful local practice. The closure question is not whether such structures are useful. It is whether they are primitive, derived, bookkeeping, or ontological.

Foundational closure is therefore a principle of primitive minimization, not a ban on primitives. It asks that primitive structure be exposed, sorted, and assigned a reconstruction task. The taxonomy below identifies four recurring ways in which explanatory work is placed outside what the closed-world account has justified.

3 The Cost of Rejecting Closure

Rejecting the closure requirement is coherent. A framework may stipulate a primitive evaluator, subsystem cut, global carrier, or repair structure without mathematical contradiction or empirical failure. It thereby locates where explanation stops.

The disagreement is not whether primitives exist, but where reconstruction ends. A closure-oriented account turns each such input into a demand for internal evaluation, admissible decomposition, compatible carrier structure, or a non-ontological repair account. A non-closure account may instead accept the input as basic. The taxonomy records that choice rather than disguising it.

4 Formal Setup

We work with four structural predicates on framework data:

I. Externalization. Primitive evaluation, state, arena, time, or measure is placed outside the system being described.

II. Artificial factorization. A subsystem decomposition is imposed as primitive rather than derived from the relational structure of the universe.

III. Premature globalization. Global carrier structure is imposed before compatibility with the relational constraints has been verified.

IV. Reification of repair. Compensating structures introduced to repair an obstruction are then treated as ontology rather than bookkeeping.

These informal descriptions orient the discussion, but the taxonomy needs predicates whose independence can be tested. The following definitions fix the formal setting.

Definition 4.1 (Closed universe)**.** *A* closed universe *is a world U in which every evaluator, clock, reference frame, and measuring device is an internal constituent of U.*

The definition does not assert that every useful physical model must represent the universe as closed. It fixes the target of this paper: once a framework claims closed-world scope, evaluators, clocks, reference frames, cuts, and repair devices may no longer be treated as external primitives without explanation.

Remark 4.2 (Operational role of closure)**.** *The preceding sections state the methodological motivation. The formal role of closure is narrower: it fixes which primitive inputs count as reconstruction burdens once a framework is given closed-world scope. The criterion below is therefore not a ban on local idealizations, but a test for whether quantities used in a universal-scope account still depend on external evaluators, imposed cuts, or non-dynamical representational choices.*

Definition 4.3 (Closed-world admissibility)**.** *A quantity Q defined on U is* closed-world admissible *if (i) Q is invariant under the internal symmetries specified by the framework, (ii) Q is independent of inadmissible cut choices used to partition U into subsystems, and (iii) Q is independent of representational choices — coordinates, gauge sections, foliations — that are not themselves dynamical degrees of freedom of U.*

The three clauses in Definition 4.3 are not meant as independent metaphysical postulates. They record three structurally distinct ways a closed-system quantity can fail to be determined by the closed system itself. If Q changes under an internal symmetry, then it depends on a description rather than on the internal state of U. If Q depends on an inadmissible subsystem cut, then it presupposes a partition not derived from the relational structure of U. If Q depends on a non-dynamical representational choice, then it imports coordinate, gauge, or foliation data not supplied by U. The criterion is therefore deliberately conservative: it requires invariance only under changes that, by the framework's own lights, do not change the closed world being described. Its role is not to rank all possible physical idealizations, but to set the standard for foundational closure: when a framework claims to describe the universe as a closed whole, whatever remains dependent on an external evaluator, an imposed cut, or non-dynamical representational data has not yet been derived from that whole.

Definition 4.4 (Primitive, derived, bookkeeping, and ontological structure)**.** *Within a structural framework datum, a structure is* primitive *when it is stipulated as an independent input to the account. It is* derived *when its use is fixed by internal relations, invariance conditions, or reconstruction criteria supplied by the framework. It is* bookkeeping *when changing or eliminating it leaves the closed-world content invariant. It is* ontological *when the framework treats it as part of what the world contains rather than as a representational device.*

This definition is operational rather than metaphysically exhaustive. It is meant to locate reconstruction burdens. If a Hilbert space, spacetime, gauge structure, probability measure, or entropy functional is recovered from internal constraints, then it has been derived relative to those constraints. If the same object is merely renamed in another vocabulary while retaining the same independent status, then it remains primitive. Borderline cases are expected; they are precisely where the closure program asks for sharper reconstruction criteria. The axes are therefore not merely classificatory labels: they mark the locations at which a closed-world account must reconstruct, eliminate, or explicitly conditionalize the primitive inputs on which it relies.

Definition 4.5 (Externalization)**.** *A framework* externalizes *if it postulates a primitive evaluation map*

$$\mathrm{Eval} : S \times O \to V,$$

whose values are not determined by relations among internal constituents of U*.*

Definition 4.6 (Artificial factorization)**.** *A framework* artificially factorizes *if it posits a primitive decomposition* $U = A \otimes B$ *(or* $U = A \cup B$*) not derived from the relational structure of* U*.*

Definition 4.7 (Premature globalization)**.** *A framework* prematurely globalizes *if it assigns global structure — for example, a Hilbert space, smooth manifold, probability measure, or global time parameter — before proving that the relational constraints of* U *are compatible with that structure.*

Definition 4.8 (Reification of repair)**.** *A framework* reifies repair *if, after an obstruction is encountered, it introduces a compensating structure and then treats that structure as primitive ontology rather than bookkeeping.*

Definition 4.9 (Structural framework datum)**.** *A* structural framework datum *is a tuple*

$$\mathfrak{F} = (U, \mathrm{Eval}, \Pi, \Gamma, \mathcal{R}),$$

where U *is a closed universe,* Eval *is either absent or an evaluation map as in Definition 4.5,* Π *is either absent or a primitive subsystem decomposition,* Γ *is either absent or a package of global carrier structure, and* $\mathcal{R}$ *is a family of compensating structures.*

Remark 4.10 (Why this datum)**.** *Definition 4.9 represents the structural roles relevant to closed-world admissibility; it does not claim that five symbols exhaust every feature of a physical theory. The entry* U *fixes the closed world under discussion. The remaining entries record the places where primitive structure can enter before it has been derived from* U*: evaluative structure* Eval*, subsystem structure* Π*, global carrier structure* Γ*, and compensating repair structure* $\mathcal{R}$*. Thus the datum tracks sources of closed-world inadmissibility. A critic who proposes an additional primitive input can sharpen the objection by specifying whether it is a new evaluative, decompositional, carrier, or compensatory role, or whether the framework datum must be enlarged by a new kind of structural role.*

Definition 4.11 (Axis predicates on framework data). *For a structural framework datum $\mathfrak{F}$, write $I(\mathfrak{F})$, $II(\mathfrak{F})$, $III(\mathfrak{F})$, and $IV(\mathfrak{F})$ for the satisfaction of Definitions 4.5, 4.6, 4.7, and 4.8 respectively. In particular, $IV(\mathfrak{F})$ requires that some element of $\mathcal{R}$ be promoted to primitive ontology rather than retained as bookkeeping.*

Remark 4.12 (Direct and compensatory input). *The first three axes concern structural input introduced directly: evaluation data, subsystem decompositions, and global carrier structure. The fourth concerns structural input introduced secondarily as a compensator for an obstruction. It is nevertheless an axis of primitive input once the compensator is assigned ontological or explanatory status. Thus the asymmetry is temporal, not taxonomic: Axes I–III classify direct primitive input, while Axis IV classifies compensatory primitive input.*

Proposition 4.13 (Relative exhaustiveness). *Let $\mathfrak{F} = (U, \mathrm{Eval}, \Pi, \Gamma, \mathcal{R})$ be a structural framework datum. Any primitive structural input represented in $\mathfrak{F}$ occurs in one of the following four components: external evaluation data* Eval*, subsystem decomposition data Π, global carrier data Γ, or compensating repair data $\mathcal{R}$. Consequently, relative to the class of structural framework data in Definition 4.9, any further axis must either be definable from one of these four components or require an enlargement of the framework datum itself.*

Proof. By Definition 4.9, the non-universe entries of $\mathfrak{F}$ are exactly Eval, Π, Γ, and $\mathcal{R}$. These entries respectively encode external evaluation, primitive subsystem decomposition, imported global carrier structure, and compensating repair structure. Therefore any primitive structural input represented by a datum of this form is represented in one of those four places. A proposed fifth independent axis cannot be an additional component of such a datum unless the datum is enlarged; if it is not an additional component, it must be a predicate defined from the four existing components. □

5 Independence Theorem

The definitions above separate four kinds of structural input. The next step is to show that this separation is not merely verbal: the four predicates can be varied independently on the stipulated class of framework data.

Theorem 5.1 (Logical independence of the four axes). *On the class of structural framework data, the predicates I, II, III, and IV are logically independent in the full Boolean sense. Equivalently, for every subset $A \subseteq \{I, II, III, IV\}$ there exists a structural framework datum $\mathfrak{F}_A$ such that exactly the predicates in A hold on $\mathfrak{F}_A$. Consequently no nontrivial Boolean equation relates the four axes on this class.*

Proof. Fix a closed universe U, a primitive evaluation map Eval, a primitive subsystem decomposition Π, a package Γ of global carrier structure, and nonempty families

$\mathcal{R}_{\text{prim}}$ and $\mathcal{R}_{\text{book}}$ of compensating structures, the former designated primitive ontology and the latter bookkeeping only.

For each subset $A \subseteq \{I, II, III, IV\}$ define

$$E_A = \begin{cases} \text{Eval}, & I \in A, \\ \varnothing, & I \notin A, \end{cases} \qquad P_A = \begin{cases} \Pi, & II \in A, \\ \varnothing, & II \notin A, \end{cases}$$

$$G_A = \begin{cases} \Gamma, & III \in A, \\ \varnothing, & III \notin A, \end{cases} \qquad R_A = \begin{cases} \mathcal{R}_{\text{prim}}, & IV \in A, \\ \mathcal{R}_{\text{book}}, & IV \notin A. \end{cases}$$

Set

$$\mathfrak{F}_A = (U, E_A, P_A, G_A, R_A).$$

By Definitions 4.5–4.11, $I(\mathfrak{F}_A)$ holds exactly when $E_A = \text{Eval}$, $II(\mathfrak{F}_A)$ holds exactly when $P_A = \Pi$, $III(\mathfrak{F}_A)$ holds exactly when $G_A = \Gamma$, and $IV(\mathfrak{F}_A)$ holds exactly when $R_A = \mathcal{R}_{\text{prim}}$. Thus the truth set of (I, II, III, IV) on $\mathfrak{F}_A$ is precisely A.

All 2^4 Boolean truth assignments are therefore realized on the class of structural framework data. If a nontrivial Boolean equation related the four predicates, at least one truth assignment would be excluded. Since none is excluded, the predicates are logically independent in the full Boolean sense. □

Corollary 5.2. *Any identification of two distinct axes loses information: some structural framework data distinguished by the full four-axis taxonomy become indistinguishable after the identification.*

Remark 5.3. *Theorem 5.1 is not a no-go theorem for a particular physical theory and does not claim that the four axes are dynamically independent in nature. It is a representation theorem for the chosen class of framework data: it shows that the four predicates cannot be collapsed into fewer Boolean predicates without losing distinctions already present at the level of structural input.*

6 Physical Content of the Independence Theorem

Theorem 5.1 is a theorem about framework data, not about dynamical decoupling inside a fixed physical theory. Its role is modest: it records which structural inputs can be varied independently in the representation class before a dynamics is specified. A single theory may contain several axes at once; another may remove one axis while leaving the others intact. The theorem shows that these possibilities are realized within the framework class. It is therefore best read as a representation lemma for the taxonomy, not as a no-go theorem about nature.

This matters because foundational discussions often compress distinct structural inputs into a single diagnosis. Measurement, time, gauge, subsystem attribution, and irreversibility are frequently described as if they were all forms of observer involvement, all forms of background dependence, or all forms of representational redundancy. The independence theorem rules out that compression for the present framework class. External evaluators, primitive subsystem cuts, imported global carrier structure, and reified repairs are independent inputs. Identifying any two of them makes some framework data indistinguishable.

The theorem therefore supplies a formal bridge from the definitions to the physical taxonomy, while leaving the physical assignments as interpretive claims. Each application below asks which of four questions is doing foundational work: where evaluation is performed, where a subsystem cut is fixed, where global carrier structure is imported, and which repair devices are promoted to ontology. Different foundational problems answer these questions differently. Consequently, the measurement problem is distinct from the problem of time, gauge-fixing obstructions are distinct from subsystem-cut problems, and irreversibility is not exhausted by the presence of an observer.

The minimality claim is correspondingly precise and internal to the chosen framework class. Any taxonomy using fewer than four independent axes must identify at least two of the structural inputs above. By the corollary, such an identification loses information about that class of data. This is why the taxonomy below is organized by axes before it is organized by named foundational problems. The axis-by-axis catalogue records the primitive structural input first; the later problem assignments then propose how familiar debates combine those inputs.

7 Taxonomic Assignments

This section turns the formal result into a diagnostic table. For each axis, it lists representative episodes in which the corresponding structural input does explanatory work and cites literature where the same pressure appears. The assignments are diagnostic rather than historical: they do not assert that a theory is invalid, that a calculational practice is illegitimate, or that the cited authors endorsed the ontology being criticized. They are also not exclusive; a single episode may instantiate several axes at once. Each assignment records which structural input must be derived or shown harmless for the framework to satisfy closed-world admissibility.

7.1 Axis I: Externalization

Axis I collects cases in which evaluative, kinematical, or representational data are treated as externally fixed.

I.1 External evaluator postulate. Axis I.1 consists in representing measurement by an external evaluation map $\mathrm{Eval}(\mathrm{state}, \mathrm{setting}) = \mathrm{outcome}$. In a closed universe, closed-world admissible evaluation must be represented by relations among internal loci.

This axis captures a structural feature of the Copenhagen interpretation; it does not dismiss its empirical success. Bohr's reply to EPR emphasized the dependence of quantum-mechanical description on the experimental arrangement [11], and Heisenberg treated the cut between observer and observed as movable but unavoidable within the standard interpretive framework [29]. Von Neumann's projection postulate provides the standard formal expression for state update associated with measurement [47]. Wigner's friend thought experiment [50] and its modern extensions [23] sharpen the point: when agents model one another inside a single closed quantum description, additional single-world consistency assumptions can become mutually incompatible. Everettian and decoherence-based approaches [21, 57] are treated here as major attempts to reformulate measurement without a primitive external evaluator.

I.2 Global state as absolute object. The structural assumption is that the state exists as an absolute element of reality: $x \in X$, $|\psi\rangle \in \mathcal{H}$, $\phi \in \mathcal{F}$, $g_{\mu\nu} \in \mathcal{G}$. A closed world has no canonical state independent of internal relational context.

Einstein's EPR paper [20] introduced a criterion of physical reality framed in terms of predictability without disturbance. In the present taxonomy, that criterion is read as pressure toward an absolute state assignment independent of the measuring procedure. Bell's theorem [10] demonstrated that local hidden-variable theories cannot reproduce all of the EPR-type correlations predicted by quantum mechanics.

I.3 External frame via coordinate or gauge choices. The structural risk is to treat coordinate and gauge choices as harmless conventions while interpreting choice-dependent results as absolute features of U. In a closed universe, conventions are internal structure; results that depend on them are not closed-world admissible.

Mach's critique of Newton's absolute space [37] identified an early form of this pressure in classical mechanics. Kretschmann showed in 1917 that general covariance does not eliminate background structure [33]: any theory can be written in generally covariant form. Anderson's absolute objects formalism [5] was a systematic attempt to identify which structures in general relativity remain external anchors despite formal covariance. Norton's analysis of the hole argument [39] sharpened the distinction between general covariance, manifold point identification, and genuine background independence. The lesson is not that standard general relativity contains a fixed background metric, but that covariance alone does not remove all surplus representational structure.

I.4 External time parameter as primitive. The structural assumption is to postulate t as a primitive and write dynamics as ∂_t laws, Hamiltonian flow. Global time is an externalization.

Dirac's Hamiltonian analysis of constrained systems [18] supplied the general framework. In canonical general relativity, the ADM formulation [2] makes the total Hamiltonian a sum of constraints, so that on shell one has $H_{\text{total}} \approx 0$. There is no external time to generate dynamics. DeWitt [15] derived the Wheeler-DeWitt equation $H\Psi = 0$ by quantizing this constraint, producing a timeless equation whose interpretation has remained contested since. Isham's comprehensive review [31] catalogs the major strategies for recovering time internally and the conceptual difficulties they face. Barbour's program [7] is a sustained argument that time should not be treated as an external primitive. Anderson's monograph [6] documents the breadth of the problem. Axis I.4 names one of quantum gravity's most widely acknowledged foundational pressures; no agreed resolution exists.

7.2 Axis II: Artificial factorization

II.1 Subsystem split treated as natural. The structural assumption is an objective decomposition $A|B$ as primitive input. A subsystem split is extra structure; observables produced from a split are conditional on the cut and are closed-world admissible only when the cut itself is derived or shown to be irrelevant.

Zanardi showed that the decomposition of a quantum system into subsystems is basis-dependent: there is no canonical tensor product structure in a generic Hilbert space [55]. In algebraic quantum field theory, the primary objects are local observable algebras attached to spacetime regions rather than a preferred global tensor-factor split into "system" and "environment" [28]. This provides a standard example of a formalism in which locality is not identified with a primitive global system–environment factorization.

II.2 Environment freezing produces artificial absolutes. The framework replaces the rest of U by fixed boundary conditions, fixed clocks, fixed reference frames, or fixed vacua. Results obtained in this way are conditional on the frozen environment and are therefore not closed-world admissible as fundamental quantities.

Caldeira and Leggett's influential model of quantum Brownian motion [13] describes a system coupled to an explicit dissipative environment, which in the present taxonomy is an instance of replacing part of the closed system by environmental data. A chosen vacuum state in quantum field theory can similarly function as a fixed anchor: changes of vacuum (Bogoliubov transformations, Unruh effect [46]) can yield observer-dependent particle descriptions — a manifestation of Axis II.2 when the choice is treated as primitive.

II.3 Pairwise reduction of multi-point structure. The structural assumption is that interaction reduces to binary primitives. Multi-way compatibility constraints are not determined by any single pair in isolation; the first nontrivial invariants are loop or triangle constraints.

Mermin's GHZ argument [24, 38] exhibited genuinely multipartite constraints that are not visible in any single pair taken in isolation. Abramsky and Brandenburger's sheaf-theoretic analysis of contextuality [3] proved that even overlap-compatible local assignments need not extend to a single global assignment.

II.4 Open-system split elevated to ontology. The structural assumption is that open-system dynamics — noise, decoherence channels, Lindblad operators — gives a fundamental description of reality. Openness models interaction with, or ignorance about, the rest of a closed universe; it cannot serve as primitive closed-world ontology without an account of the omitted degrees of freedom.

The Lindblad equation [35] characterizes Markovian completely positive semigroup dynamics for an open subsystem. Treating it as a fundamental equation of motion imports an effective environment or coarse-graining assumption, which is the present instance of Axis II.4.

7.3 Axis III: Premature globalization

III.1 Background carrier space assumed. The structural assumption is a manifold M taken as a given carrier. In a closed relational world, "where" cannot be primitive without additional structure.

Mach [37] argued against Newton's absolute space. Earman and Norton [19] showed through the hole argument that manifold point identification can carry surplus representational structure not fixed by observable fields. Background independence is treated in the quantum-gravity literature as a central unfinished issue [45].

III.2 Global Hilbert space as primitive. The structural assumption is a single global Hilbert-space representation whose linear superposition structure is taken as primitive.

Haag's theorem [27] shows that the interaction-picture idealization of quantum field theory — which requires a unitary equivalence between free and interacting representations — fails in the standard infinite-volume setting. That theorem does not by itself settle the status of renormalized quantum field theory, but it does show that the unreconstructed global-Hilbert-space representation has nontrivial mathematical consequences. Perturbative renormalization can then be read as a repair strategy for working around this failure, rather than as a direct theorem-level consequence of Haag's result. Haag's algebraic quantum field theory program [28] is a sustained attempt to formulate the theory in terms of local algebras and their representations rather than a single preferred global interaction-picture Hilbert-space representation.

III.3 Global time foliation treated as given. General relativity has no preferred foliation of spacetime.

In canonical gravity, this appears as the absence of any preferred decomposition into spatial slices plus an external time parameter [2, 34, 31]. The ADM formalism [2] introduces lapse and shift functions to encode freedom in the choice of foliation rather than to select a preferred one. In the present taxonomy, treating a particular foliation as primitive would be an instance of Axis III.3. Kuchař's analysis [34] identifies the choice of internal time variable as the central underdetermined freedom in canonical quantum gravity.

III.4 Infinitesimalization before integrability. Infinitesimal gauge-fixing does not extend globally.

Singer's theorem [44] gives the mathematical form of this obstruction: for non-abelian Yang-Mills theory on compact Euclidean spacetime, no gauge condition defines a global section of the gauge-orbit space. The Gribov ambiguity [26] is the corresponding physical manifestation: standard gauge conditions can intersect a single gauge orbit more than once, so the Faddeev-Popov procedure fails globally. This is the structural pressure captured by Axis III.4: a gauge condition may be locally consistent but globally obstructed.

III.5 Probability as primitive measure. In standard quantum mechanics, the Born rule is postulated as a primitive probability rule.

Gleason's theorem [25] characterizes additive probability measures on the closed subspaces of a Hilbert space, thereby underwriting the Born-rule form on projectors, but it does not derive the Hilbert space itself. Everett [21] formulated quantum mechanics in terms of relative states, thereby shifting the problem of probability away from an explicit collapse postulate. Deutsch [16] and Wallace [48] developed decision-theoretic derivations of probability in the Everettian setting. Frauchiger and Renner [23] sharpened the measurement-theoretic tension in nested-observer scenarios under additional consistency assumptions.

III.6 Global smooth structure at bedrock. Differentiability and PDE structure are assumed at the foundational level.

Penrose's twistor program [40] recasts spacetime structure in terms of twistor space and complex-geometric data rather than ordinary spacetime points. Regge calculus [42] discretizes spacetime to avoid importing smooth structure as primitive. Connes's noncommutative geometry [14] replaces the smooth manifold with a spectral triple derived from algebraic data. Each of these programs can be read as a response to Axis III.6, without agreement on a resolution.

7.4 Axis IV: Reification of repair

IV.1 Gauge data reified as dynamical substances. A gauge-theoretic framework introduces redundancy to maintain comparison consistency, then risks treating gauge-dependent connection data, gauge-invariant field strength, and loop or holonomy observables as if they had the same primitive status.

The Aharonov-Bohm effect [4] showed that electromagnetic potentials can affect interference even when the local field strength vanishes on the relevant region, motivating a careful distinction between gauge-dependent potential, gauge-invariant field strength, and global holonomy data. Wu and Yang [54] identified holonomy — the path-ordered exponential around a closed loop — as a gauge- invariant carrier of physical information. Axis IV.1 is not the use of these distinct objects, but the failure to keep straight which are gauge-dependent bookkeeping devices and which encode invariant content.

IV.2 Curvature reified as a fundamental field. The metric is often treated as primitive geometric data, while curvature is then granted independent ontological weight beyond its status as structure derived from the metric or connection.

Weyl's original unified theory [49] treated electromagnetism geometrically through a generalized comparison structure. The Kobayashi-Nomizu formulation [32] of connections on principal bundles gives the standard differential- geometric setting in which curvature is associated with connection data on a bundle. In the present taxonomy, the remaining question is whether the bundle and connection are derived from internal comparison structure or introduced as primitive carrier data.

IV.3 Global gauge-fixing assumed possible. The structural assumption is that a global gauge section can be chosen without obstruction.

Singer's theorem [44] and the Gribov ambiguity [26] show that this assumption fails in the standard non-abelian Yang-Mills settings where Gribov copies occur. The standard perturbative treatment uses Faddeev-Popov ghosts [22] to implement gauge fixing locally, introducing further repair structures — ghost fields — associated with the handling of gauge redundancy. Each layer of repair (BRST symmetry [9], Batalin-Vilkovisky formalism [8]) is a further compensating structure whose ontological status must be controlled.

IV.4 Irreversibility as fundamental. Entropy production is treated as ontic rather than as a coarse-grained or effective description.

Boltzmann's H-theorem [12] obtained a monotonicity result from reversible Newtonian mechanics using the Stosszahlansatz (molecular chaos assumption). Loschmidt's reversibility objection [36] and Zermelo's recurrence objection [56] showed that the monotonic conclusion requires an assumption not derivable from the reversible base theory alone. Boltzmann accordingly reformulated the conclusion in statistical terms. Prigogine's program [41] pursued a derivation of irreversibility from reversible dynamics. These difficulties do not prove that openness is the only available source of time-asymmetric behavior, but they do show that standard derivations require extra assumptions — coarse-graining, typicality, special initial data, or openness — beyond reversible microdynamics itself. Axis IV.4 arises when one such repair is then reified as fundamental.

IV.5 Anomalies treated as unexplained constraints. Anomalies appear; the theory then imposes cancellation conditions as consistency requirements whose deeper structural origin may remain unexplained.

The Adler-Bell-Jackiw anomaly [1] showed that the classical axial current conservation is broken by quantization. The standard response treats anomaly cancellation as a consistency condition on the matter content. Witten's global $SU(2)$ anomaly [53] showed that an odd number of Weyl fermion doublets makes the fermion determinant change sign under a large gauge transformation, yielding a consistency restriction on admissible matter content. In the present taxonomy, such cancellation requirements are treated as imposed consistency restrictions unless their origin is derived from prior structural data.

IV.6 Renormalization group reified as running physics. Wilson's renormalization group [51] provides a powerful framework for relating effective descriptions across scales. Axis IV.6 occurs when the renormalization group flow, represented by scale-dependent couplings, is treated as independently fundamental rather than as a relation among effective descriptions. The renormalization group is a map between such descriptions at different resolution scales. In the present taxonomy, renormalization is associated with Axis III.6 when continuum smooth structure is imposed before its global closed-world admissibility has been established.

8 Case Study: Measurement as Externalization

The measurement problem is the simplest place to see the taxonomy work as an instrument rather than as a list. In the textbook scheme, a quantum state evolves unitarily until a measurement occurs. At that point an outcome is registered and the state is updated relative to the measurement result. The formalism is extraordinarily successful, but from the closed-world standpoint it contains an evaluative boundary: a system is described on one side, and an apparatus, observer, or outcome map is treated as already available on the other. This is Axis I in its direct form.

In the notation of Definition 4.5, the textbook scheme effectively supplies an evaluation map from state and setting to outcome. Nothing in the bare unitary dynamics of the closed composite system identifies a unique outcome event with the same status as the pre-measurement evolution. The closure burden is therefore not to deny measurement practice, but to account for how evaluation is represented inside the world being described.

Decoherence changes the location of the burden. By coupling a system to an environment, decoherence explains the suppression of interference in a preferred basis and clarifies why some reduced descriptions become stable for practical purposes. In the present taxonomy, this is genuine progress: part of what looked like a primitive classical apparatus is absorbed into physical dynamics. But decoherence normally relies on a system–environment split, a choice of relevant observables, and a trace over omitted degrees of freedom. Thus an Axis I burden is partly converted into Axis II structure. The evaluator is less external, but the admissibility of the cut remains to be explained if the account is to be closed-world in scope.

Everettian approaches pursue a stronger internalization strategy. They retain unitary dynamics universally and treat measurement outcomes as branch-relative rather than as the result of an externally imposed collapse. In closure terms, this is an attempt to remove the primitive evaluator by making observers and records internal to the quantum state. The residual burden is not the same as in the textbook scheme. It concerns the status of branch decomposition, probability, and the relation between internal records and experienced outcomes. The axis has shifted: the external evaluator is not simply assumed, but the factorization and measure structures used to articulate branching must still be justified.

Relational approaches make a different move. They deny that state assignments are absolute and treat outcomes as relative to interacting systems. This directly attacks the assumption that there is a single view from outside all interactions. In the present taxonomy, relational quantum mechanics is therefore a natural closure-oriented strategy: it attempts to internalize evaluation by making it a relation among physical systems. The remaining question is whether the network of relative facts can be made globally coherent without reintroducing a privileged evaluator, cut, or carrier structure.

The case study illustrates the point of the four-axis scheme. Progress on measurement is not measured only by whether the word "observer" is removed. A proposal may internalize evaluation while leaving a subsystem cut primitive; it may explain the cut while importing a carrier structure; it may repair an obstruction by adding a new device whose ontological status is unclear. The taxonomy records these burden transfers. It does not declare the measurement problem solved, but it shows what closure-oriented progress would have to achieve: an account of outcomes, records, and probabilities whose evaluative, decompositional, and representational resources are internal, derived, or explicitly conditional.

9 Why Closure Generates Explanatory Compression

The main explanatory payoff of the Closure Program is compression. Before the closure criterion is applied, several foundational tensions look structurally unrelated. Measurement appears to concern observers and outcomes. The problem of time appears to concern Hamiltonian constraints and temporal ordering. Gauge theory appears to concern redundant variables and global sections. Irreversibility appears to concern entropy, coarse-graining, and time-asymmetric behavior. Anomalies and renormalization appear to concern consistency conditions and scale-dependent descriptions. These are genuinely different physical problems. The claim is not that they have one solution or one mechanism.

The closure criterion reveals a common form of explanatory dependence. In each case, some structure needed by the local formalism is doing work whose closed-world status has not yet been secured. Measurement uses an evaluator or outcome map. Statistical mechanics uses a coarse-graining, typicality measure, or system–environment split. Gauge theory uses sections, ghosts, or compensating complexes to manage redundancy. Quantum field theory uses carrier representations and renormalization structures to relate descriptions across regimes. Canonical gravity removes external time and thereby exposes the need for internal temporal structure.

Closure compresses these cases because it asks the same question in each domain: where is the explanatory resource located relative to the closed world? The answer falls into four roles. The resource may be an external evaluator, an imposed cut, an imported carrier, or an ontologized repair. Those roles are not interchangeable, as Theorem 5.1 records. But they are comparable: each marks a way in which explanatory structure is placed outside what has been internally justified.

This is not merely a relabeling of familiar disputes. A relabeling would attach new names to old problems without changing their relations. The closure taxonomy changes the relations among the problems. It shows, for example, why removing an observer from quantum measurement does not by itself solve the problem of subsystem factorization; why eliminating a gauge redundancy does not by itself produce an internal time; why deriving a local carrier representation does not by itself settle the status of repair structures; and why irreversibility cannot be reduced simply to the presence of an observer. Progress along one axis leaves the other reconstruction burdens visible.

The compression is therefore diagnostic and constructive. It is diagnostic because it identifies the primitive input on which a foundational tension depends. It is constructive because it states what would count as progress: internalize the evaluator, derive the cut, reconstruct the carrier, or demote the repair to bookkeeping. The taxonomy does not solve these tasks. It makes their non-equivalence and their recurrence explicit.

10 Foundational Problem Assignments

The preceding section sorted structural inputs by axis. Standard foundational problems usually combine several inputs in one physical setting. This section therefore reassembles the axis catalogue into assignments for canonical foundational problems. The assignments are not parallel analogies or exhaustive descriptions of the corresponding literatures. They are claims about structural dependence relative to Definition 4.3, not claims that the listed topics have no other physical or interpretive content. Each assignment identifies the structural input that carries foundational weight within the present formal criterion and states the reconstruction problem produced by that input. They are therefore non-exclusive: a topic may involve several axes, and the listed axes name the primitive inputs whose removal would change the closed-world status of the problem. This convention avoids treating familiar problem names as mutually exclusive bins.

Measurement problem. The measurement problem is assigned primarily to Axis I.1 (Bohr [11], von Neumann [47], Everett [21], Wigner [50]). The standard measurement scheme separates an evolving quantum system from an evaluating apparatus or observer and then represents the outcome by a map from state and setting to value. That map is not generated by the same unitary dynamics assigned to the closed system. It is supplied at the boundary between system and evaluator. In the present taxonomy, that boundary is the structural input: it is an external evaluator in the sense of Definition 4.5.

By Theorem 5.1, this structural input is separable from the inputs involved in global smoothness, global gauge fixing, or renormalization. The measurement problem can be formulated without a gauge bundle, a continuum limit, or a curvature repair; conversely, removing a gauge-fixing obstruction does not remove the measurement cut. The reconstruction problem is therefore broader than choosing an interpretation of the quantum state: it is to give a closed-world account of evaluation itself: outcomes must be represented by internal relations among constituents of U, not by a primitive map from the outside. Collapse, Everettian branching, decoherence, Wigner-friend analyses, and relational quantum mechanics [43] are classified here as strategies for removing, relocating, or internalizing this evaluator.

Problem of time. The problem of time is assigned to the conjunction of Axes I.4 and III.3 (Dirac [18], DeWitt [15], Isham [31], Kuchař [34], Anderson [6]). Ordinary Hamiltonian dynamics begins with an external parameter t and represents evolution by a derivative or flow with respect to that parameter. Canonical general relativity does not preserve this structure: the Hamiltonian is constrained, and the Wheeler-DeWitt equation $H\Psi = 0$ contains no external time variable. The external time parameter has therefore been removed in the closed-system formulation.

The independence theorem separates two inputs that are often conflated. Axis I.4 is the externalization of time: the assumption that there is a primitive temporal parameter relative to which the universe evolves. Axis III.3 is premature globalization: the attempt to equip the whole closed system with a single global time structure before proving that the relational constraints admit one. A local or approximate clock does not by itself supply a global time for the universe. The reconstruction problem is consequently more precise than the undifferentiated demand to recover time. One must specify internal degrees of freedom that can function as clocks, prove that the resulting ordering is invariant under the constraints, and determine when such local temporal orderings assemble into a global parameter. The multiplicity of strategies surveyed by Isham and Anderson is therefore structurally expected: removing external time and constructing global time are independent tasks.

Status of gauge symmetry. The status of gauge symmetry is assigned to the conjunction of Axes IV.1 and III.4 (Dirac [18], Henneaux–Teitelboim [30], Singer [44], Wu–Yang [54]). Gauge variables are introduced to make local comparisons consistent; they encode how descriptions at neighboring points or regions are to be compared. The issue is not the use of gauge variables, which is often indispensable, but the promotion of gauge-dependent comparison data to primitive ontology. That is Axis IV.1. The Aharonov-Bohm [4] and Wu-Yang [54] analyses show why the distinction is subtle: connection data, field strength, and holonomy are related, but they do not have the same invariant status.

Axis III.4 enters because local gauge fixing does not imply global gauge fixing. Singer's theorem and the Gribov ambiguity show that a gauge condition that works locally in the space of fields need not define a global section. Thus the foundational issue is twofold: gauge descriptions are redundant, and the local repair cannot be globalized without obstruction. By Theorem 5.1, this obstruction is not reducible to the measurement problem or to subsystem attribution. A gauge theory may contain a global gauge-fixing obstruction without an external-observer problem in the relevant sense; conversely, a measurement problem may arise without non-abelian gauge copies. The reconstruction problem is to identify the invariant content of the theory — for example in holonomy or algebraic terms — without reifying the repair variables and without assuming a global section where none exists.

Origin of irreversibility. The origin of irreversibility is assigned to the conjunction of Axes IV.4 and II.2 (Boltzmann [12], Loschmidt [36], Prigogine [41]). The standard microscopic dynamics used in these discussions is time- symmetric. Standard derivations of irreversibility require additional asymmetry-bearing assumptions, often in the form of environment freezing (Axis II.2), coarse-graining, or special initial data. Entropy production may then be treated as fundamental (Axis IV.4). Loschmidt's objection shows that irreversibility does not follow from time-symmetric dynamics without such further structure. The assignment is not the claim that every irreversible phenomenon literally contains an external environment; it is the claim that derivations of macroscopic arrow-like behavior often depend on a primitive cut between retained and discarded degrees of freedom, or on a compensating entropy principle whose status is not derived from the closed dynamics. The reconstruction problem is to obtain the relevant arrow from internal dynamics and admissible coarse descriptions rather than from an imposed partition or reified thermodynamic repair.

Charge quantization. Charge quantization is assigned here to the conjunction of Axes III.1 and III.6. Standard derivations proceed through topological arguments about the global bundle structure of gauge fields or through Dirac's monopole argument [17, 54]. On this assignment, the topology that quantizes charge enters through global bundle structure together with global smoothness; the closed-world question is whether those global structures are derived or imported. The issue is therefore distinct from the status of gauge variables as repair devices: charge quantization may be expressed in a gauge-theoretic language, but the structural input doing the work here is the global carrier structure on which the topological argument relies. The reconstruction problem is to show how the relevant bundle or smooth global data arise from closed relational constraints rather than being assumed as background structure.

Anomalies. Anomalies are assigned to the conjunction of Axes IV.1, III.4, and IV.5 (Adler-Bell-Jackiw [1], Witten [53]). Gauge symmetry is introduced to encode comparison consistency (Axis IV.1), gauge-fixing may fail non-perturbatively (Axis III.4), and the resulting obstructions may appear as unexplained cancellation constraints (Axis IV.5). This assignment separates three questions that are often compressed: whether gauge variables are ontology, whether local gauge choices globalize, and whether anomaly-cancellation conditions are derived or imposed. The reconstruction problem is to explain anomaly cancellation, when it occurs, as a consequence of the closed framework rather than as an external consistency constraint added after quantization.

11 The Axes as Open Problems

Once the assignments are made, each primitive input becomes a reconstruction task. The question is no longer only which axis is present, but what would have to be derived for that axis to disappear from the primitive data. We state the non-redundant open problems explicitly.

Problem I.1. Give a mathematically precise formulation of measurement in a closed universe that does not postulate an external evaluator, an external state space, or an observer-system cut not derived from the internal structure of the universe.

Problem I.2. Characterize physical states in a closed universe without treating an absolute state space as primitive. Under what conditions can Hilbert-space state vectors be recovered from internal relational data?

Problem I.3. Give a criterion for closed-world admissibility of a physical quantity that is independent of coordinate choice, gauge section, and foliation. Characterize which quantities in general relativity and gauge theory satisfy this criterion.

Problem I.4. Derive the temporal succession of events in a closed universe from internal relational data, without postulating a global time parameter. What theorem-level conditions on internal structure are sufficient, or necessary, for a global time parameter to exist?

Problem II.1. Characterize the admissible decompositions of a closed universe into subsystems in a way that does not depend on an externally imposed cut. What physical invariants, if any, survive changes among admissible cut choices?

Problem II.3. Identify the minimal arity at which genuinely new multi-point constraints appear that are not determined by pairwise data. Give a systematic classification of such constraints.

Problem III.2. Derive the Hilbert space structure of quantum mechanics from more primitive relational data, without postulating a global vector space. What internal structure of a closed relational system supports or reconstructs linear superposition?

Problem III.4. Give a systematic classification of the obstruction to global gauge-fixing for non-abelian gauge theories. Singer's theorem [44] establishes existence of the obstruction. The present paper does not attempt a complete classification of its structure.

Problem III.5. Derive the Born rule from internal indistinguishability or symmetry data of a closed system, without postulating a probability measure.

Problem IV.1. Give a formulation of gauge theory in which the gauge potential does not appear as a primitive variable. Characterize the invariant content of gauge theories, for example in terms of holonomy or algebraic data, without reference to a chosen gauge section.

Problem IV.2. Derive the metric and curvature of spacetime from more primitive relational or combinatorial data, without postulating a smooth manifold as background.

Problem IV.4. Derive the arrow of time from the internal dynamics of a closed reversible system, without postulating an asymmetric environment or an entropy primitive. Give a theorem-level statement of conditions under which irreversibility appears as a derived rather than fundamental feature.

No separate problem statements are given for Axes II.2, II.4, III.1, III.3, III.6, IV.3, IV.5, and IV.6. In the present organization, II.2 and II.4 are treated as subsystem-environment specializations of Problem II.1; III.1 and III.6 as geometric versions of Problem IV.2; III.3 as a temporal specialization of Problem I.4; IV.3 as the global form of Problem III.4; IV.5 as downstream from Problems IV.1 and III.4; and IV.6 as downstream from Problems III.2 and IV.2. The list above therefore records only the non-redundant open problem statements.

12 Synthesis: The Compressed Core

The formal theorem, the axis catalogue, the problem assignments, and the open reconstruction tasks can now be compressed into a single structural pattern.

> **Within the closure program, foundational tensions arise when frameworks externalize, factorize, or globalize before verifying relational integrability, and then reify the compensators introduced to manage the resulting obstructions.**

This statement is not an additional thesis appended to the taxonomy; it is the taxonomy written in one sentence. Each item in Section 7 is a domain-specific instance of the same pattern, and the assignments in Section 10 treat standard foundational problems as different combinations of its four independent components.

In this precise sense, the common thread is externalization. Axis I is externalization in the direct sense: evaluation, time, state, or frame is placed outside the closed system. Axes II–IV are not reducible to Axis I, but each externalizes explanatory work in a distinct way: a subsystem cut is fixed outside the relational structure, a global carrier is imported before compatibility is established, or a repair device is promoted before its bookkeeping status has been secured. The four-axis taxonomy records these different modes of placing structure outside what has been internally justified.

13 Evaluation Criteria

The synthesis also fixes how the proposal should be judged. Because the paper offers a structural taxonomy rather than a new dynamics or interpretation, its evaluation criteria are internal to that aim. The taxonomy is evaluated by three criteria. First, it separates axes that are logically independent: no axis is a Boolean consequence of the other three, by Theorem 5.1. Second, it identifies, for each axis, the relevant closed-world datum that must be supplied rather than assumed: an internal evaluator, an admissible subsystem decomposition, compatible global carrier structure, or a non-ontological account of repair devices. Third, it converts familiar foundational debates into reconstruction problems whose success conditions are explicit. Thus the taxonomy is not assessed by whether it supplies a new interpretation of quantum mechanics, a new quantization of gravity, or a new dynamics for irreversible processes. It is assessed by whether the four predicates are well-defined, independent, and useful for locating the primitive structural input at issue in each case.

These criteria make the taxonomy contestable as a structural proposal. A successful objection may do one of three things: exhibit a fifth structural input independent of the four axes, show that one of the four axes is reducible to the others on the class of framework data, or give a closed-world reconstruction of one of the listed cases while retaining the corresponding primitive input. Absent such an objection, the taxonomy gives a minimal four-axis account of the structural inputs considered here, relative to the class of framework data in Definition 4.9.

The applications below do not depend on treating successful local calculations as defective. They depend on a sharper distinction: a device may be legitimate for calculation while remaining inadmissible as primitive ontology in a closed universe. Gauge choices, subsystem cuts, external clocks, probability measures, coarse-grainings, and repair fields are frequently necessary in practice; the foundational question is whether their use is derived, invariant, and cut-independent, or whether it is imported as unexplained structure.

14 Scope of the Closure Requirement

The closure requirement is not a general rule that every scientific explanation must eliminate all primitives. Ordinary theory construction often begins with Hilbert spaces, manifolds, probability measures, boundary conditions, clocks, gauges, and coarse-grainings whose status is not derived in the model at hand. Nothing in the taxonomy treats that practice as defective. The issue is the change of status that occurs when a framework is presented not merely as an effective model within a domain, but as a foundational account of the closed world. At that point the same structures can no longer function both as unexplained external input and as part of the explanation of the whole. This last sentence is the methodological commitment of the paper; it is not a theorem forced by the formalism.

This is why closed-world admissibility is stricter than ordinary physical usefulness. Entropy may be meaningful relative to a coarse-graining; particle number may be meaningful relative to an observer or asymptotic structure; entanglement may be meaningful relative to a factorization. Such quantities are not rejected because they are contextual. The question is whether the context on which they depend has itself been accounted for inside the closed-world description. Context-dependence is compatible with the criterion when the context is internal, derived, invariantly specified, or explicitly left as a condition of applicability. What fails the criterion is an unaccounted context promoted to primitive foundational status.

The criterion therefore carries a definite philosophical commitment. It favors accounts in which the structures used to describe the whole are not borrowed from outside the whole. A substantivalist may instead treat some background structures as primitive constituents of reality; an effective realist or pluralist may decline the demand for a single closed-world reconstruction. Those are coherent ways to reject the closure program. They do not, however, remove the distinction the taxonomy tracks. They relocate it: what the present framework treats as a reconstruction problem is instead accepted as primitive input. Thus the taxonomy should not be read as metaphysically neutral. It is a map of the burdens generated by one closure-oriented research program.

The same limitation applies to the formal theorem. Theorem 5.1 does not show that the four axes are dynamically independent in nature, nor that physical mechanisms cannot connect them. It shows that the four roles cannot be identified inside the stipulated class of framework data without losing information. The physical assignments in Section 10 therefore remain structural diagnoses, not theorem-level derivations from the dynamics of the theories cited. Their force is comparative: they say which primitive input would have to be derived, eliminated, or made explicitly conditional for the corresponding closed-world tension to disappear.

Finally, the terms *primitive*, *derived*, *bookkeeping*, and *ontological* are used operationally within this framework. A structure is primitive when it is stipulated as an independent input to the account. It is derived when its use is fixed by internal relations, invariance conditions, or reconstruction criteria supplied by the framework. It is bookkeeping when changes in that structure do not alter the closed-world content. It is ontological when the framework treats it as part of what the world contains. Borderline cases are expected; they are precisely the cases in which the reconstruction burden has not yet been discharged.

15 Conclusion

The argument now runs in one direction: state the closure criterion, separate four structural inputs, prove their independence on the chosen framework data, assign physical episodes to those inputs, clarify the scope of the closure requirement, and convert the assignments into reconstruction problems. The result is a formal taxonomy of structural input for closure-oriented foundational programs in physics. Definitions 4.5, 4.6, 4.7, and 4.8 specify four predicates — externalization, artificial factorization, premature globalization, and reification of repair — on structural framework data. The precise formal result is Theorem 5.1: these predicates are logically independent on the class introduced in Section 4.

The applications propose assignments of foundational episodes to combinations of the four independent axes, record representative literature, and formulate the associated reconstruction problems. The conclusion is direct and limited to that structural claim: recurring tensions often treated as separate are organized by four kinds of input fixed prior to dynamics: primitive evaluation data, imposed subsystem cuts, imported global carrier structure, and the ontological promotion of repair devices. The unifying operation is externalization, understood broadly as the placement of explanatory structure outside what the closed-world description has itself justified. Until the open problems in Section 11 are resolved, frameworks that accept the closure premise continue, on the present analysis, to rely on structural inputs whose closed-world admissibility has not yet been established. For a companion development of the closed-system program for Axes I, II, and IV, see Wolfe's companion manuscript on diagonal redundancy and subsystem attribution [52].

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